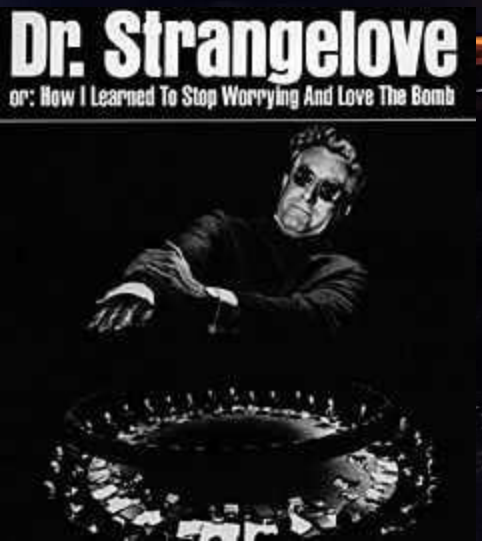
AN INTRODUCTION TO SPECIAL RELATIVITY



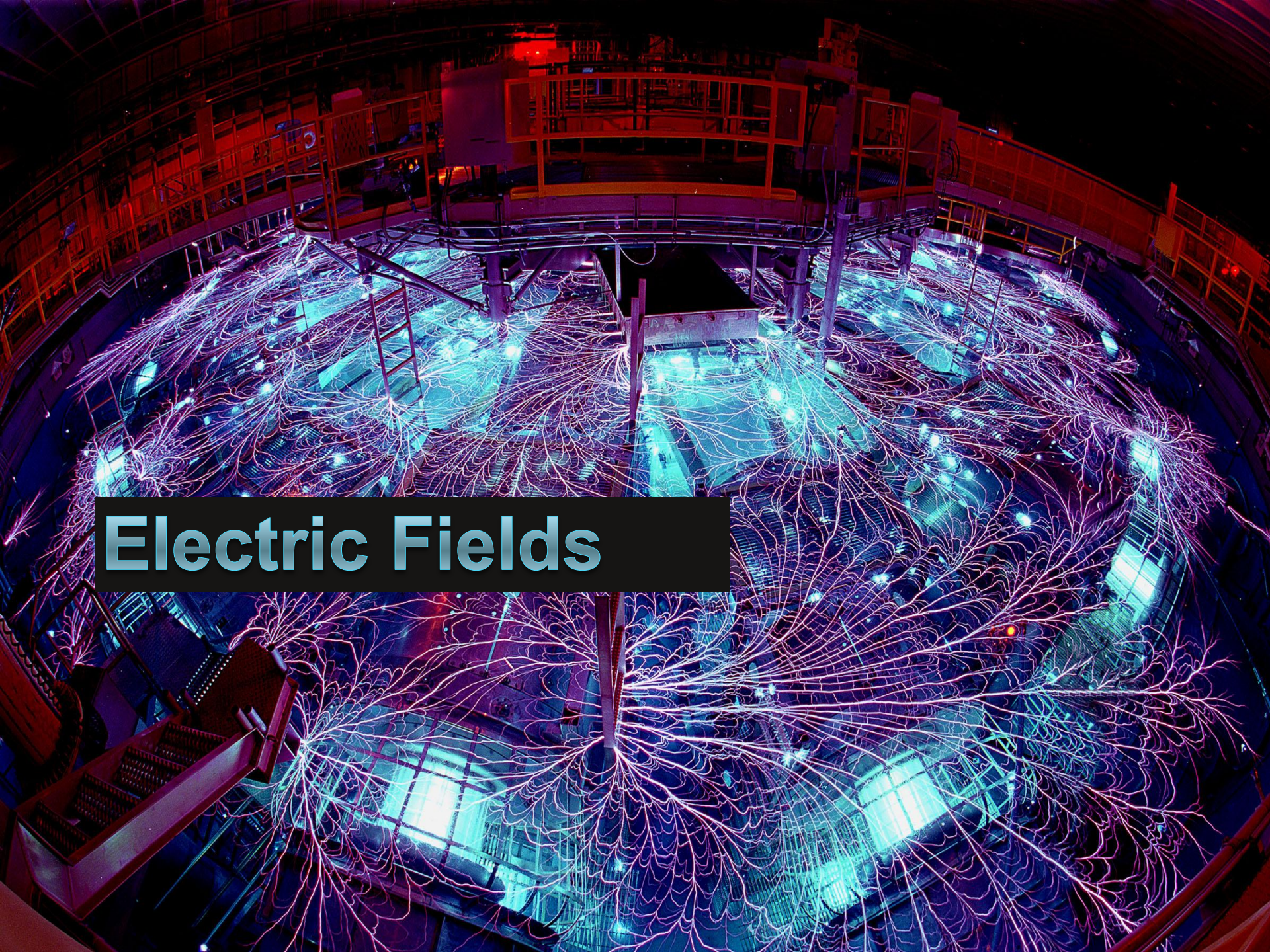
(or: How I Learned To Stop Worrying And Love The Bomb)

Outline

- Overview
- Electric Fields
- Magnetic Fields
- A Brief History of Light
- Electromagnetic Unification
- Something Rotten in Denmark
- The Many Faces of Einstein
- Special Relativity Results
- General Relativity
- Questions?

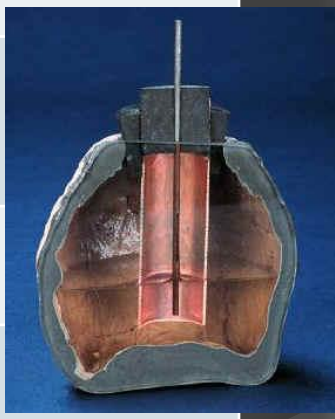
Overview

- Speed of light
  - Known finite for a few hundred years
- Cutting edge physics in early 1800's
  - Electricity, magnetism, light
- Maxwell's Equations, 1870+
  - Unifies electric, magnetic fields
  - Predicts light speed constant and invariant
- Special Relativity
  - Implied by speed of light being constant

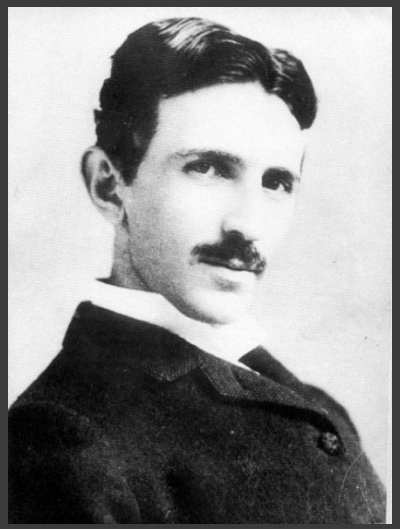

Electric Fields

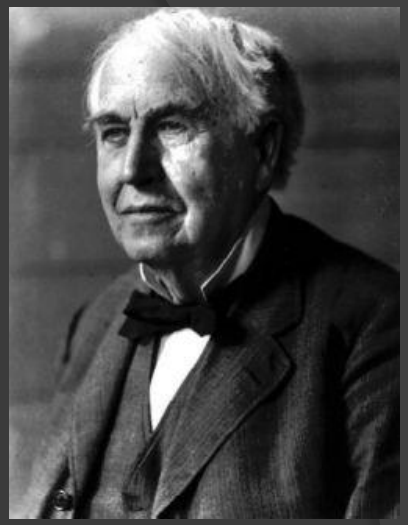
Electricity History

Date	Event	Person
600 BC	Static electricity from rubbing fur	Thales of Miletus
300 BC	Baghdad Battery	Used for electroplating?
1650	Electric and magnetic forces distinct	Cardano
1675	Electric force crosses a vacuum	Robert Boyle
1745	Charges in Leyden Jar	Leyden
1752	Lightning is electricity, positive and negative charges distinguished	Benjamin Franklin

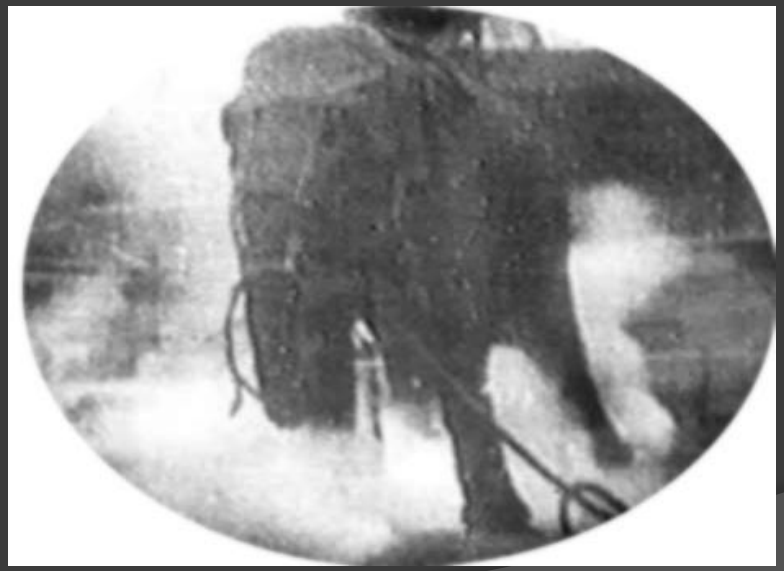


Trivia



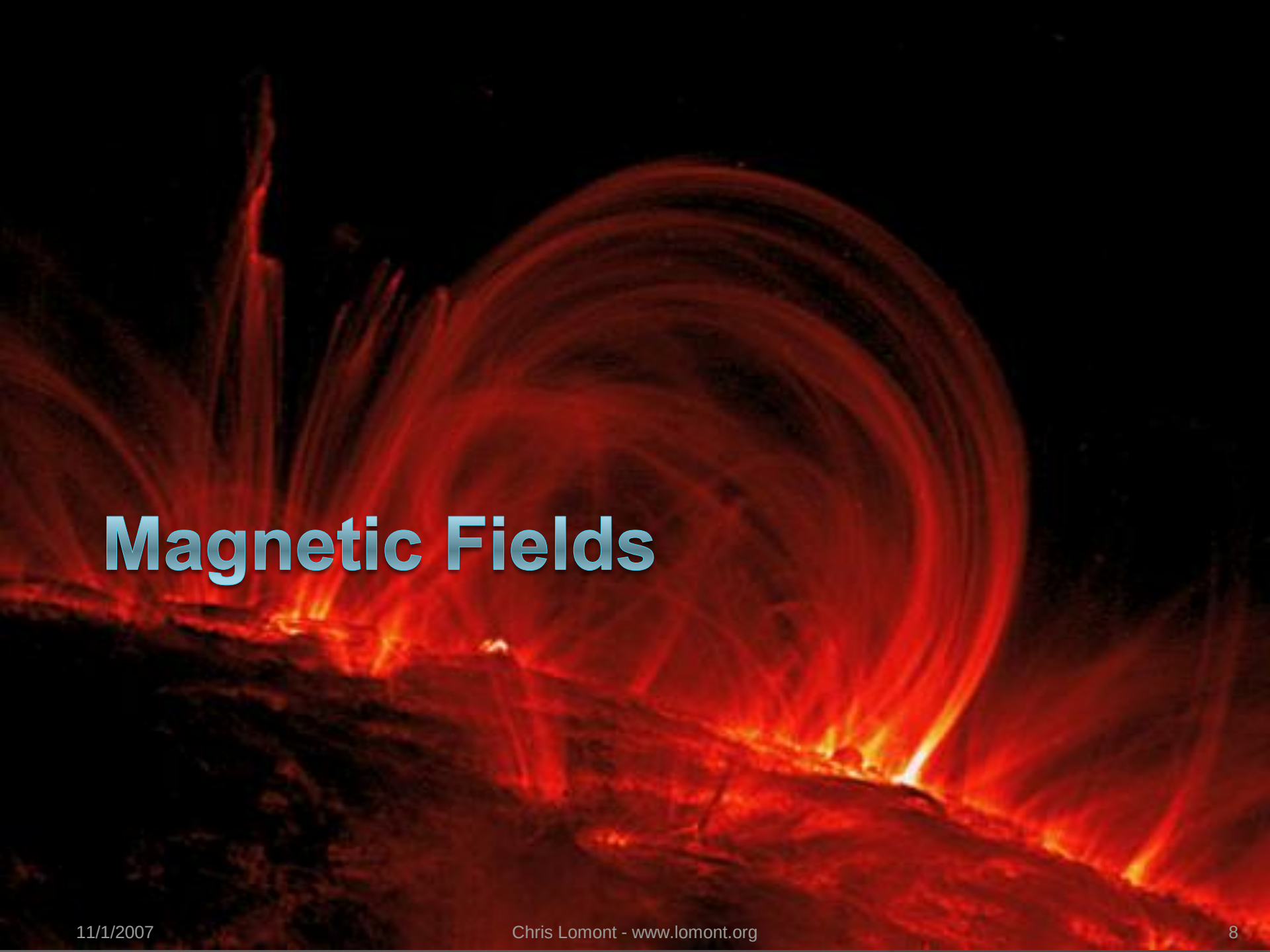


- Tesla vs Edison
  - AC vs DC
  - Edison electrocuted “Topsy” the elephant to show AC dangerous



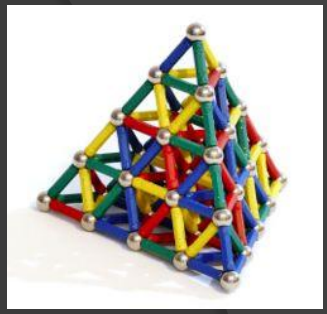
Electric field

- Gravitational analogy
  - acts on charges the way gravity acts on mass
  - Negative charged particles are “anti-mass”
- E=F/q
  - F = force
  - E = electric field
  - q = charge on test particle
- Stronger than gravity by factor of 10^{38}

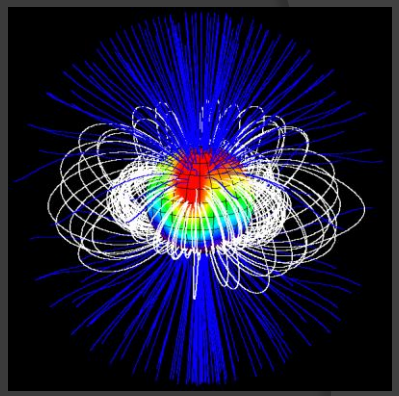


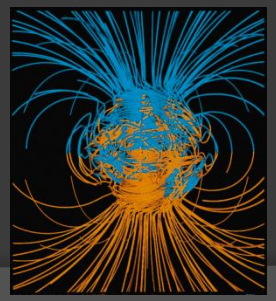
Magnetic Fields

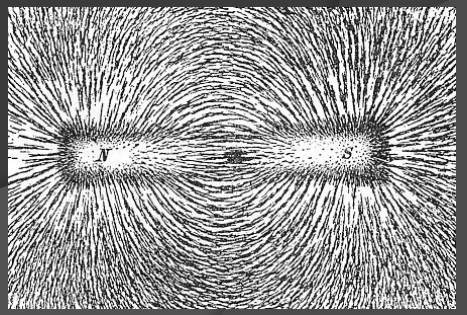
Magnetism History



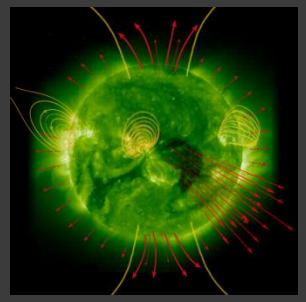
- Thales, 500 BC, lodestone
- Compass for navigation, 1100 AD
- Oersted, 1819, discovered link by accident
  - Electric current influences a needle.
- Coulomb, Ampere, Gauss, Faraday Ohm, others discover “rules”





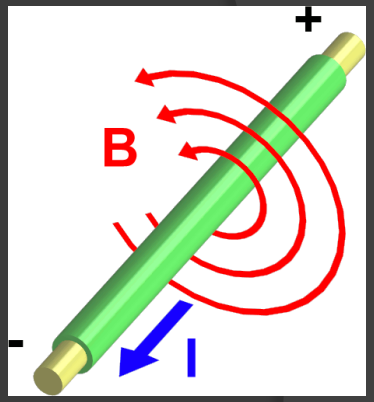


Magnetic field



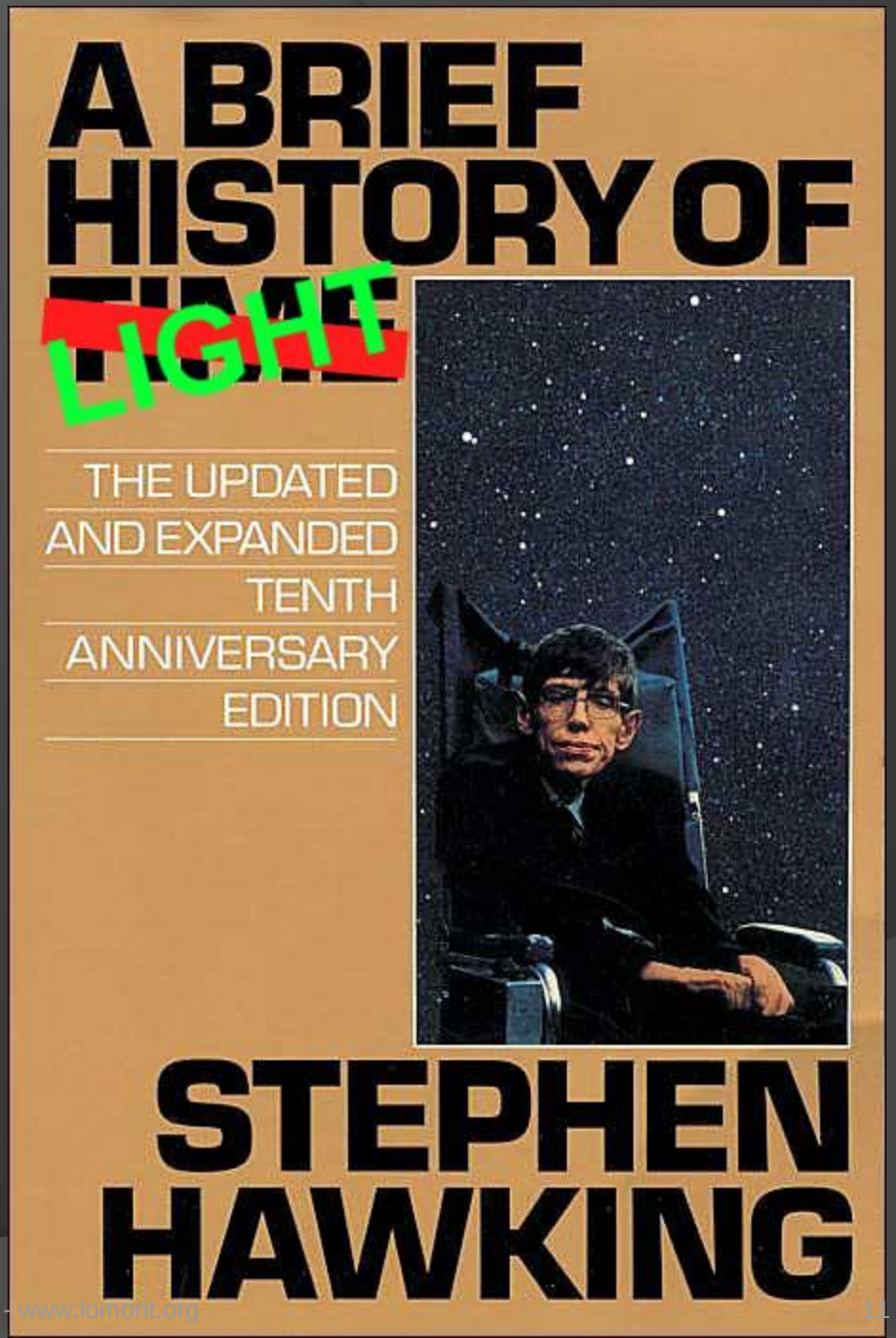
- Magnetic field **B**
  - Creates torque on small bar magnet
- Moving particle
  - F force, q charge, B magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$

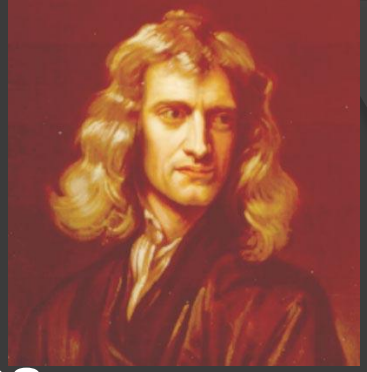


- Poles always in pairs
- Measured in Tesla (N s / C m)
  - Earth: 30-60 μT (3.0×10^{-5} T to 6.0×10^{-5} T)
  - 16 T required to levitate a frog

A Brief History of Light



Light History



- Early Indian, Greek, Hindu theories
- Descartes 1637 – light continuous substance
- Hooke - waves 1660s
- Newton, 1672, *Opticks*, light is particles
- Huygens - waves 1678
- Due to Newton, light was treated as a particle stream before 19th century
- Thomas Young - Wave nature shown in 1801
  - Fresnel (1788-1829)
  - Foucault 1850

Light speed

- Danish astronomer Ole Roemer (1644-1710) in 1676
  - Systematic variations in Io orbiting Jupiter implied light took 22 minutes to traverse Earth's orbit diameter (which was unknown then)
  - demonstrated light had a *finite speed*
- Armand Hippolyte Fizeau (1819-1896) in 1849
  - Used light through a gear over several kilometers
  - Obtained 3.1x10^8 m/s

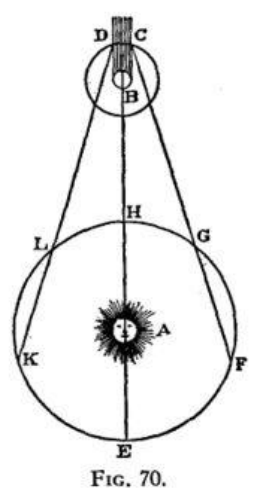
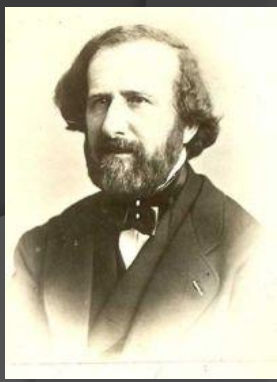
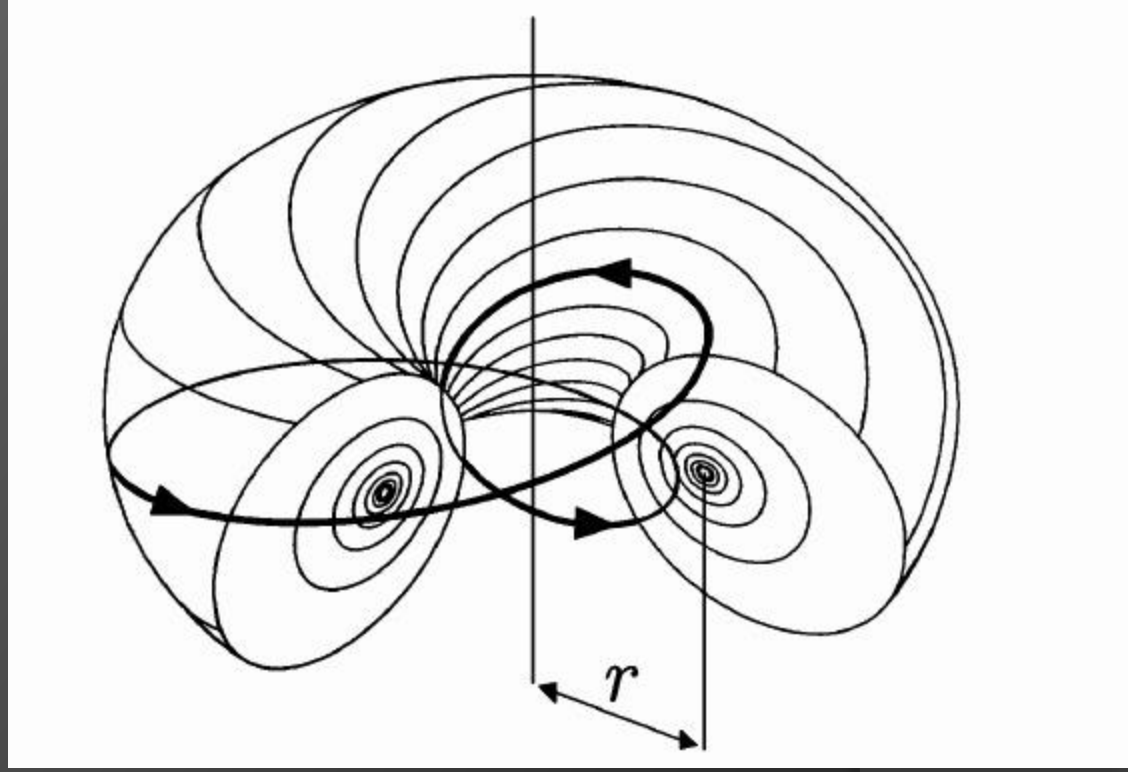


Fig. 70.





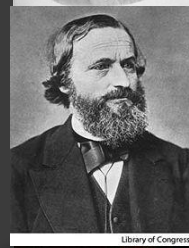
Electromagnetic Unification

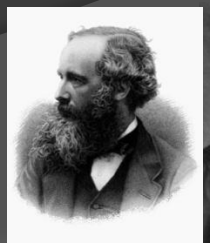
Electromagnetic Unification

- Many laws relating electric and magnetic phenomena
  - Coulomb's Law, Biot-Savart Law, Faraday's Law, Ampere's Law, Kirchhoff's Laws, Gauss's Law, Ohm's Law
- 1860
  - Time is ripe for deeper understanding.
  - Maxwell unifies all these ideas.
  - Resulting theory subsumes optics.

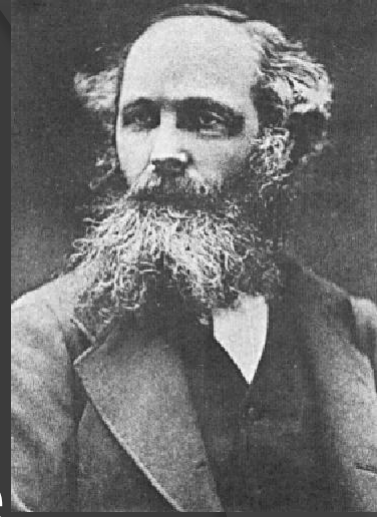







James Clerk Maxwell



- (1831-1879)
- Wrote first scientific paper at 14
- Correctly discovered how we perceive color
- Took first color photograph
  - Tartan Ribbon in 1861
  - using red, green, blue filters

Maxwell's Equations

- 4 equations, presented in 1861
- Describe interrelationship between:

Electric Field	E
Magnetic Field	B
Electric Charge	q
Electric Current	J

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

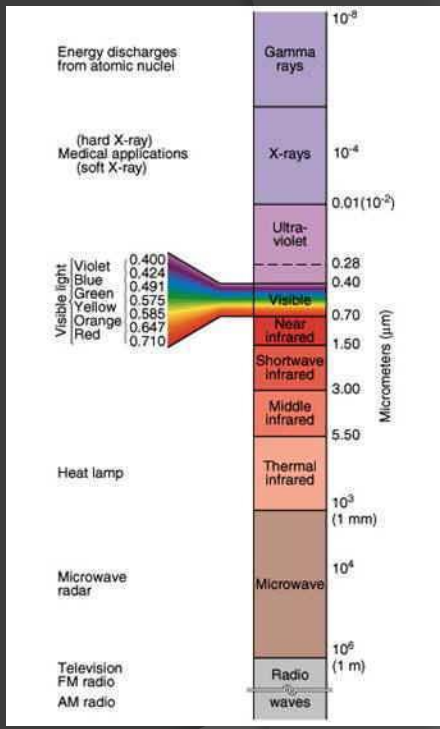
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

- Derived electromagnetic wave equation in 1865
  - Demonstrates light is an electromagnetic wave.

Experimental constants

- Need two experimental constants:
  - ε_0 - permittivity of free space
    - how well a vacuum transmits (“permits”) an electric field
      - $8.8541878176 \times 10^{-12}$ F/m
  - μ_0 – permeability of free space
    - how well a vacuum responds to a magnetic field.
      - $4\pi \times 10^{-7}$ N/A^2.



Meaning of the Equations

1. First relates change in electric flux E to sources and sinks
2. Second says no magnetic monopoles.
3. Third says as magnetic field B changes, it creates a curled electric field E
4. Fourth says as electric field E changes it creates a curled magnetic field.

- **E** = Electric vector field
- **B** = Magnetic vector field
- ρ = charge enclosed = 0 in free space
- **J** = current density = 0 in free space

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

Solution to Maxwell's Equations

- In free space (J=0, p=0):
  - Take 1, curl it, getting 2.
  - Substitute 3, identity 4, and 5
  - Obtain result

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$1. \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$2. \nabla \times \left(\nabla \times \vec{E} \right) = -\frac{\partial \left(\nabla \times \vec{B} \right)}{\partial t}$$

$$3. \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$4. \nabla \times \left(\nabla \times \vec{E} \right) = \nabla \left(\nabla \cdot \vec{E} \right) - \nabla^2 \vec{E}$$

$$5. \nabla \cdot \vec{E} = 0$$

  - Result is a wave equation, saying electric field can propagate at speed

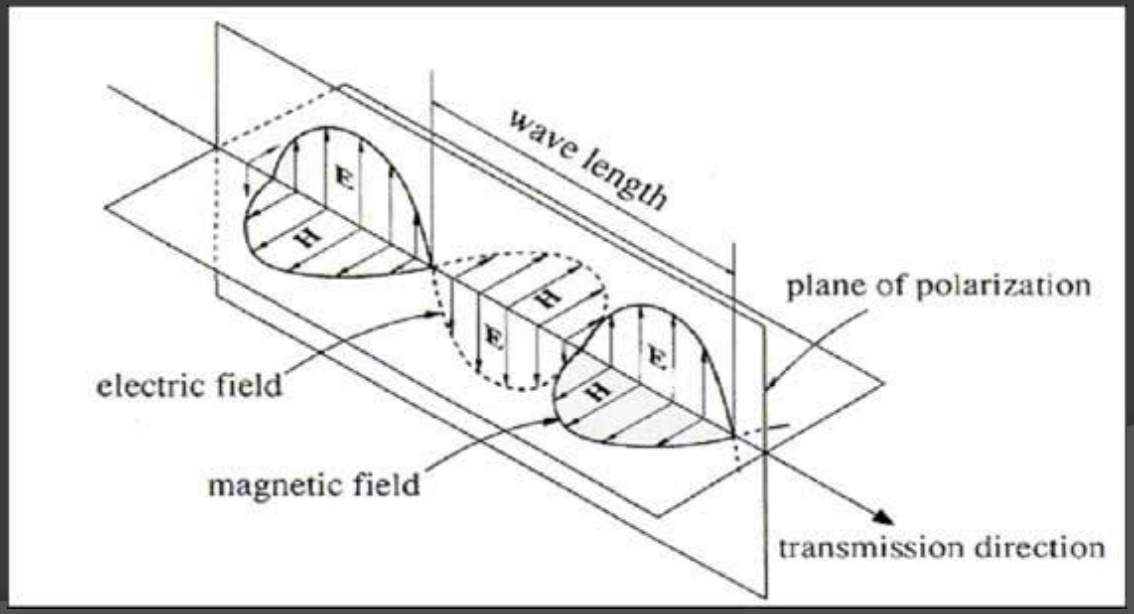
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

- Similarly, $\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$
- Both $\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ and $\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

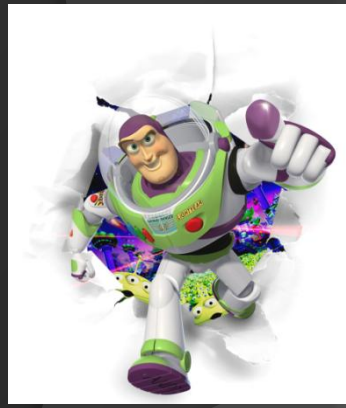
  are equations of a wave traveling at the speed $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299792458 \text{m/s}$
- Matches speed of light!
  - Based on this, Maxwell predicted light was an electromagnetic wave.

Consequences

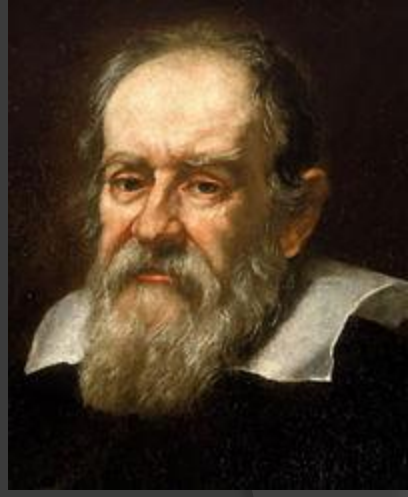
- Electromagnetic waves travel at $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
- E/B = c tells relative sizes
- Does not depend on speed of emitter!



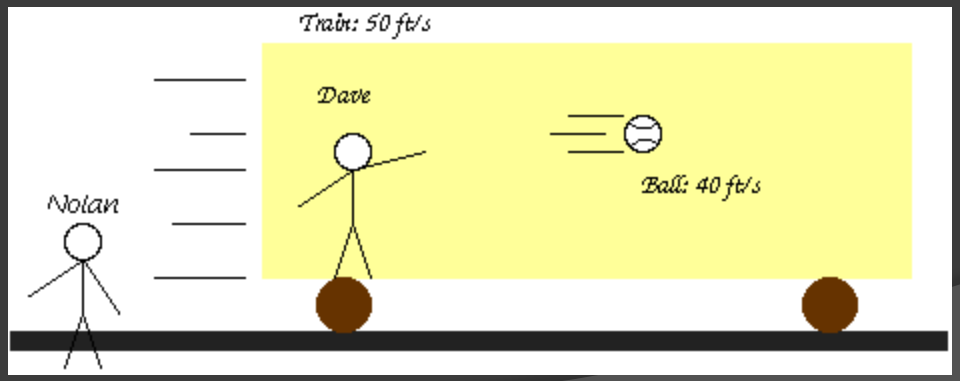
Something Rotten in Denmark



Galilean Transformations

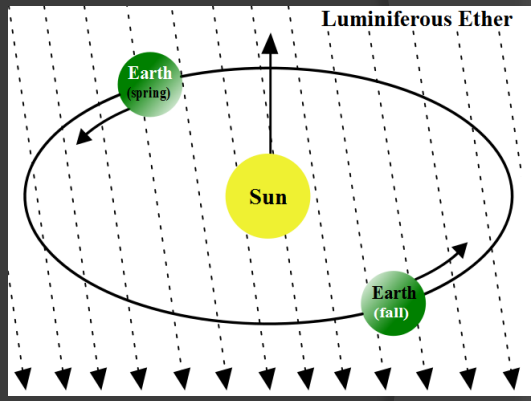


- Galileo taught velocity adds linearly
  - $v_t = v_1 + v_2$
- If on a train going 50 m/s, and you throw a ball at 40 m/s relative to the train, someone on the ground should see ball moving at 90 m/s = 40 m/s + 50 m/s



Maxwell's equations

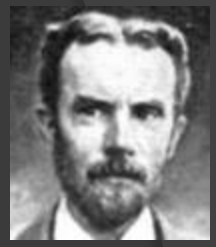
- Say light travels at the same speed no matter the speed of emitter
  - Breaks earlier intuition
- Luminiferous Ether
  - Michelson-Morley experiments 1887
- Speed of light *exactly* 299,792,458 m/s



Problems with wave/particle theories

- Wave theories
  - Broken by photoelectric effect - Hertz
    - Ejected electron independent of light intensity.
    - Explained by Einstein, 1905
      - Uses Planck's idea of quantizing energy
      - Energy of light in quantum packets called *photons*
- Particle theories
  - Young double slit breaks
    - Light behaves here as a wave

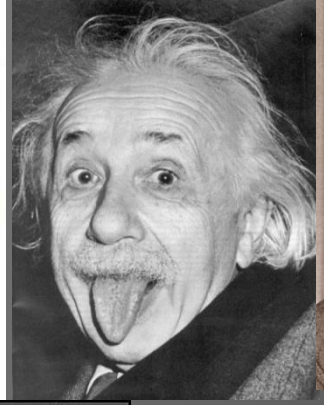
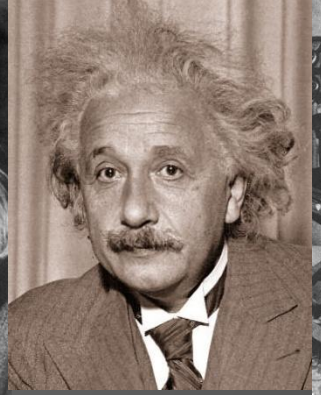
Pre-Einstein “Relativity”

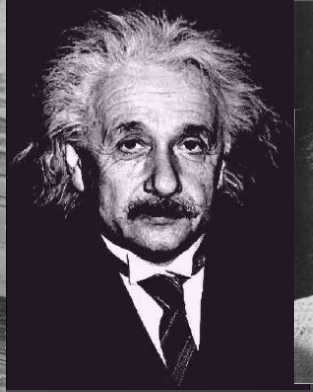
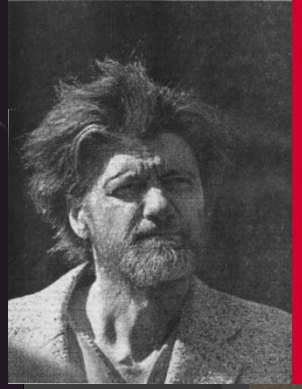
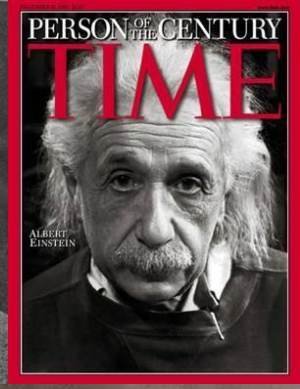


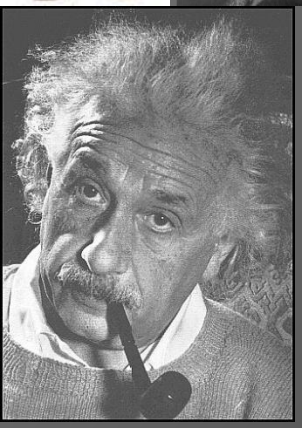
- Heaviside 1888
  - derived field contraction from Maxwell eqns
- Fitzgerald (1889, qualitatively), Lorentz (1892, quantitatively)
  - Lorentz-Fitzgerald length contraction
  - Same as special relativity, different cause
  - 1895 – time dilation also

$$\sqrt{1-v^2/c^2}$$

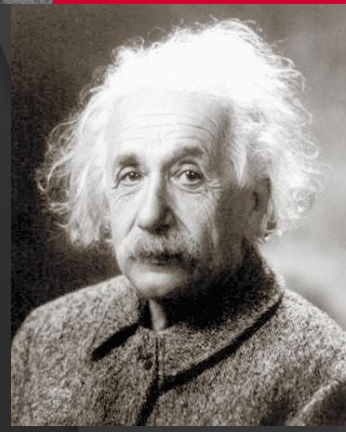
- Poincare 1898
  - Formulated principle of relativity – no experiment can discriminate between uniform motion and rest
  - Formulated special relativity in 1905, months before Einstein, under different assumptions



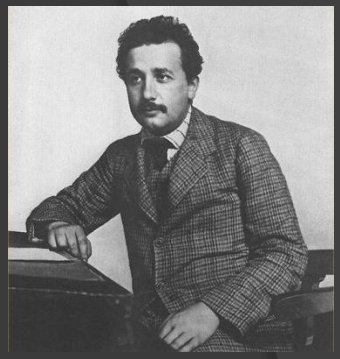
The Many Faces of Albert Einstein



Dr. Peter Venkman : “Einstein did his best stuff when he was working as a patent clerk!”

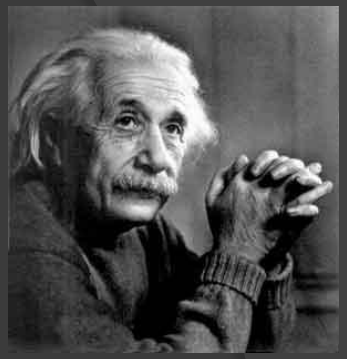
Dr. Raymond Stantz: “You know how much a patent clerk earns?!”

Einstein



- Explained special relativity under different assumptions than Lorentz, Poincare
- Postulates – in inertial frame:
  - 1st : physical laws are the same
  - 2nd : speed of light is the same
- Inertial frame is :
  - non-accelerating, non-rotating system
- Special Relativity 1905

Effects of postulates

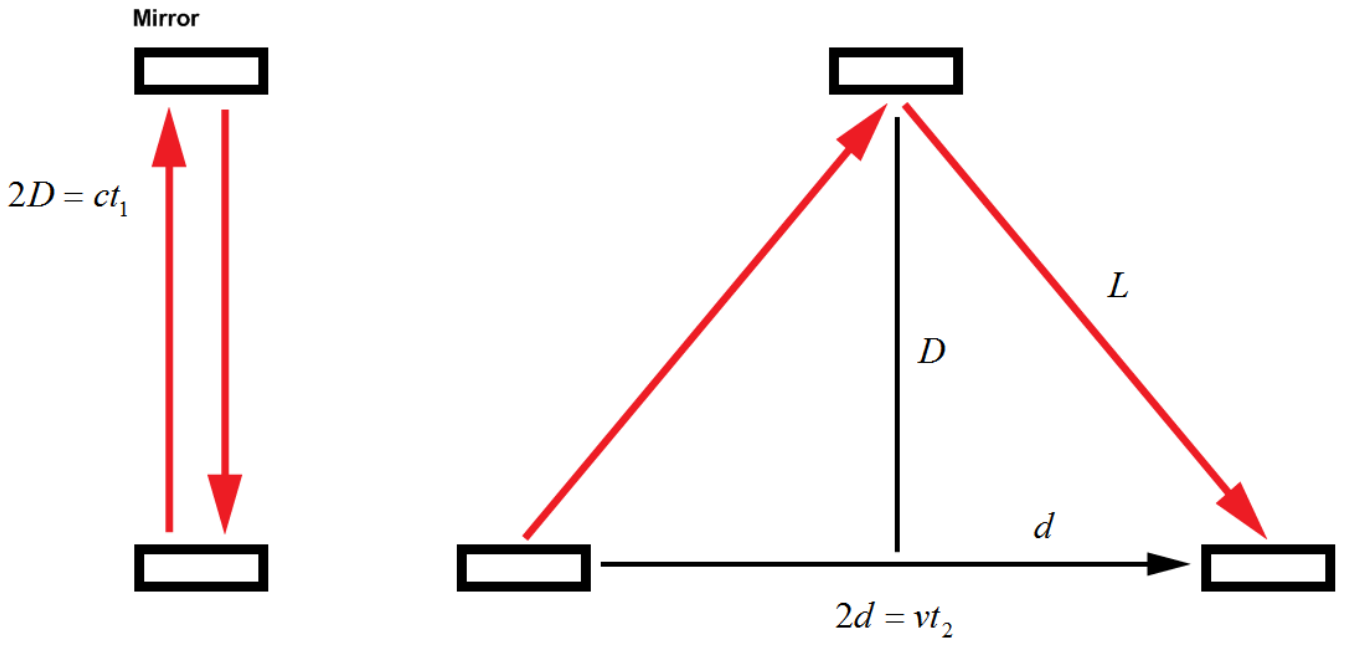


- Postulates – in inertial frame:
  - 1st : physical laws are the same
  - 2nd : speed of light is the same
- Both seem reasonable, consequences seem unreasonable
  - Time dilation
  - Length contraction
  - Mass increase
  - Equivalence of mass and energy

Einstein's Fashion Legacy

Basic Results in Special Relativity

Light Clocks



$$L = \sqrt{D^2 + d^2} = \sqrt{\left(\frac{ct_1}{2}\right)^2 + \left(\frac{vt_2}{2}\right)^2}$$

$$2L = ct_2$$

$$2\sqrt{\left(\frac{ct_1}{2}\right)^2 + \left(\frac{vt_2}{2}\right)^2} = ct_2$$

$$(ct_1)^2 + (vt_2)^2 = (ct_2)^2$$

$$t_1 = t_2\sqrt{1 - \frac{v^2}{c^2}}$$

$$t_1 = t_2\gamma$$

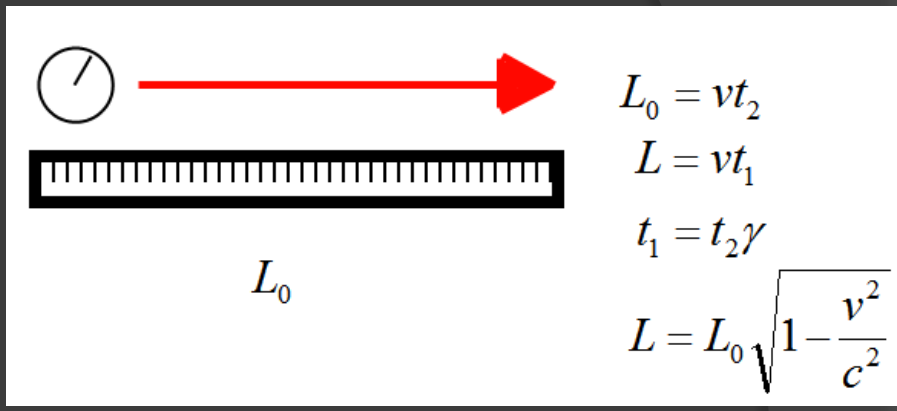
$$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

$$0 \le \gamma \le 1$$

Time dilation

- Notice the times are related by $t_1 = t_2\gamma$
  - The gamma value is $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$
  - For small velocities v, this term is close to 1 and the times are similar
  - For large velocities v close to the speed of light c, gamma is near 0 and t_2 is much larger than t_1!
- This is *time dilation*
  - *Moving clocks run slower*

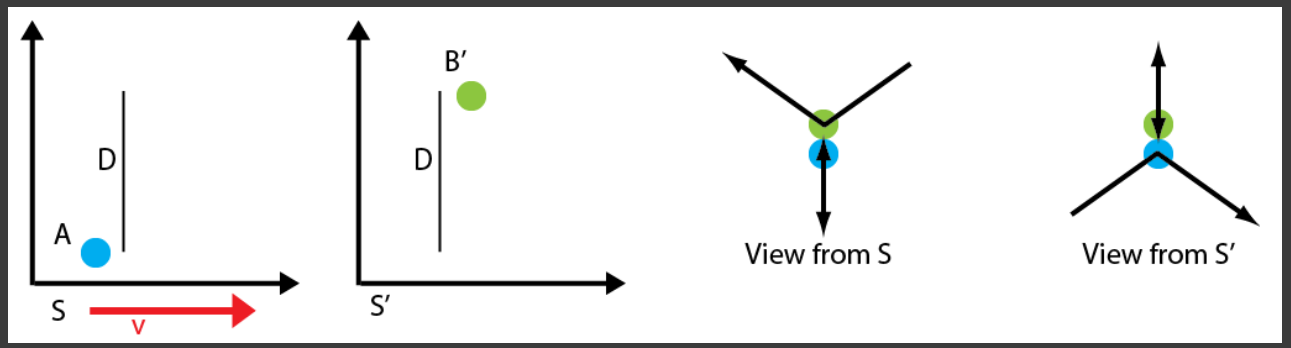
Length shortening



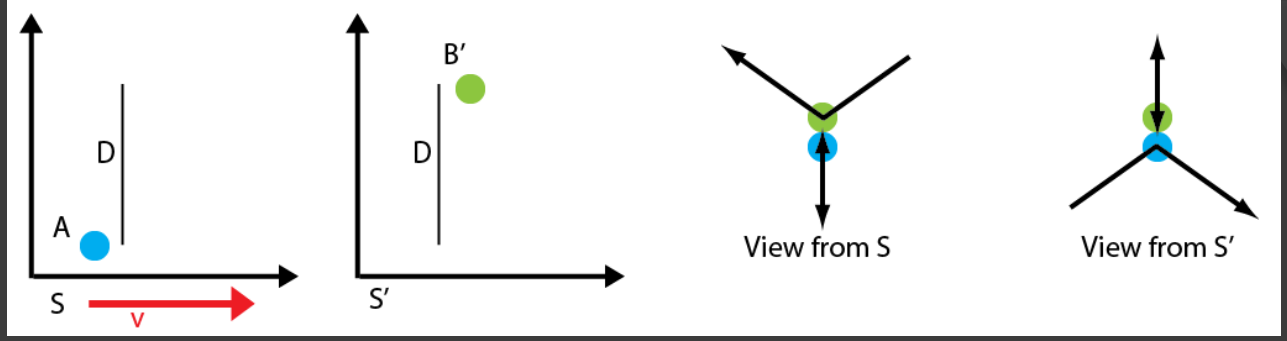
- Distance = rate x time
- Rate
  - defined as ratio to c,
  - all observers agree on value,
  - thus time dilation implies length dilation.
- Called Lorentz-Fitzgerald contraction
- This is *length contraction*
  - *Length decreases in direction of motion*

Relativistic Mass

- Conservation of momentum *in all frames* leads to relativistic mass



- Frame S moves right at velocity v
- A moves up at velocity v_A , B moves down at velocity v_B
- After impact, return to original places



- Momentum in frame S: $m_A v_A = m_B v_B$
- From S: $D = t_1 v_A$, $D = t_2 v_B$
- Time dilation: $t_1 = t_2 \gamma$
- Substitute:

$$m_A t_2 = m_B t_1$$

$$m_A t_2 = m_B t_2 \gamma$$

$$m_A = m_B \sqrt{1 - v^2 / c^2}$$

- Recall, both masses same at rest
- Conclude: *S sees increase in mass m_B*

Relativistic Mass

- Rest mass is denoted m_0
- Mass at velocity v denoted m_v
- Leads to $$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$
- This is *relativistic mass increase*
  - *Mass increases to infinity as velocity goes to speed of light c*
  - *Rest mass is smallest*

Gamma

- Relation of speed to dilation effect
- Everyday speeds have little effect

mph	factor reduced
95	0.99999999999999
300	0.9999999999999
948	0.999999999999
2999	0.99999999999
29991	0.999999999
948395	0.999999
29983389	0.999
94602117	0.99
292315012	0.9
443571206	0.75
580771037	0.5
649321759	0.25
667255121	0.1
670583097	0.01
670616626	0.0001

Relativity of simultaneity

- Breaks! Depends on observer
- Thought experiment
  - Observer in middle of solar system sees two events from opposite ends at the same time
  - Observer moving sees one first, then the other, due to speed of light being finite
  - Are the events “simultaneous” ? Who is right? Neither.

$E=mc^2$

- Force is change in momentum with respect to time.
- Classically, $F = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt} = 0 + ma = ma$ since mass is constant
- Kinetic energy is work done in bringing an item from rest to a speed v_f (at position s_f)

$$K = \int_0^{s_f} F\,ds = \int_0^{s_f} \frac{d(mv)}{dt}\,ds$$

$E=mc^2$

- Steps
  - Change variables
  - Replace mass
  - Integrate by parts

$$\int x\,dy = xy - \int y\,dx$$

$$x = v \qquad y = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$dx = dv \qquad dy = d\left(\frac{m_0 v}{\sqrt{1 - v^2/c^2}}\right)$$

  - Integrate
  - Expand, algebra
  - Replace mass

$$K = \int_0^{s_f} \frac{d(mv)}{dt} ds = \int_0^{mv_f} v\,d(mv)$$

$$= \int_0^{v_f} v\,d\left(\frac{m_0 v}{\sqrt{1 - v^2/c^2}}\right)$$

$$= \frac{m_0 v^2}{\sqrt{1 - v^2/c^2}}\Bigg|_0^{v_f} - \int_0^{v_f} \frac{m_0 v}{\sqrt{1 - v^2/c^2}} dv$$

$$= m_0 v_f^2 / \gamma_f + m_0 c^2 \sqrt{1 - v^2/c^2}\Big|_0^{v_f}$$

$$= m_0 c^2 / \gamma_f - m_0 c^2$$

$$K = mc^2 - m_0 c^2$$

$E=mc^2$

- Interpret $K = mc^2 - m_0c^2$
  - Kinetic energy is the change in relativistic mass times c^2
- Interpret total energy $mc^2 = K + m_0c^2$

  where $E_0 = m_0c^2$ is the *rest energy*
- The total energy of a moving mass is then

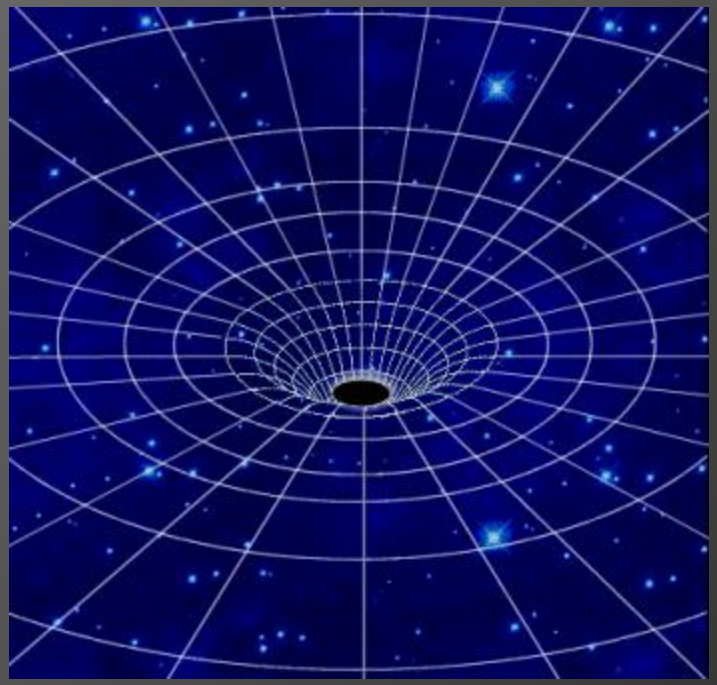
$$E = mc^2$$

Implications

- Twin paradox
  - A moving clock runs more slowly than a stationary clock, so spaceman ages weirdly.
- No global timeframe in universe.
- Energy no longer conserved, mass no longer conserved, only mass-energy conserved.
- Space-time
  - Space and time are intertwined
- Time travel, closed time-like curves.

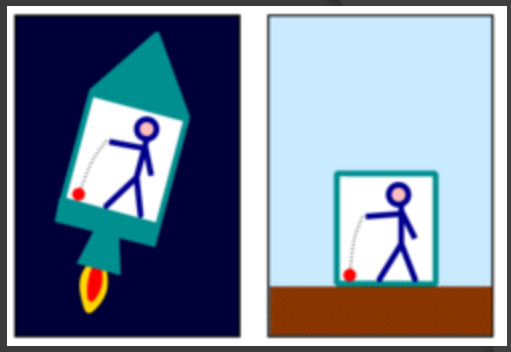
Summary

- Moving clocks slowed by motion
- Moving rods shrink the direction of motion
- Mass increases with velocity
- Energy and mass equivalent
- $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$ is factor of change

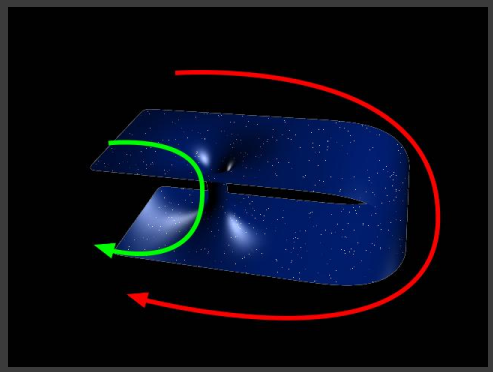


General Relativity

General Relativity



- Equivalence principle :
  - Gravity and acceleration indistinguishable
  - spacetime tells matter how to move; matter tells spacetime how to curve
- 1915/16
  - Hilbert submitted first,
  - Einstein published first

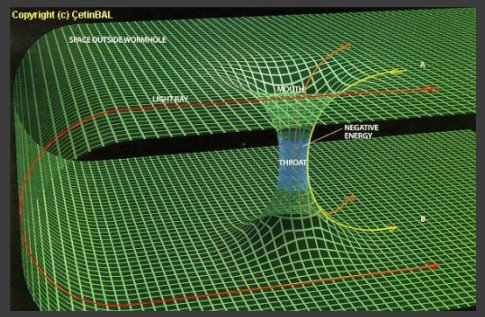


General Relativity

- One equation:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

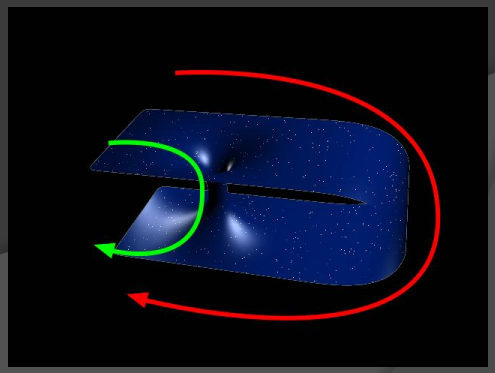
- Relates curvature of space-time to mass-energy density locally



- System of 10 nonlinear differential equations

Implications

- Black holes
  - Curvature bends light
  - 1784 – John Michell realized possible
  - 1795 – Laplace defined requirements for black hole
- Universe not static
  - Hubble, Einstein error
- Big bang
- Large scale structure of the universe
- Time travel ideas
  - Wormholes
  - Likely not possible

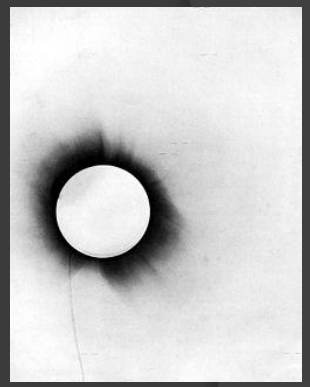


Experimental Evidence

- Mercury orbit anomaly, 1915
  - Perihelion precession of Mercury

Amount (arcsec/century)	Cause
5025.6	Coordinate (Like a top precession)
531.4	Gravitational tugs of the other planets
0.0254	Oblateness of the Sun (quadrupole moment)
42.98±0.04	**General relativity**
5600.0	Total
5599.7	Observed

Experimental Evidence



- Eddington, eclipse 1919
  - Made Einstein instant hero
- Gravitational redshift
  - Tested 1959 by Pound-Rebka experiment
- Muons
- Flying atomic clocks around the planet
- GPS
- Particle accelerators

Questions?