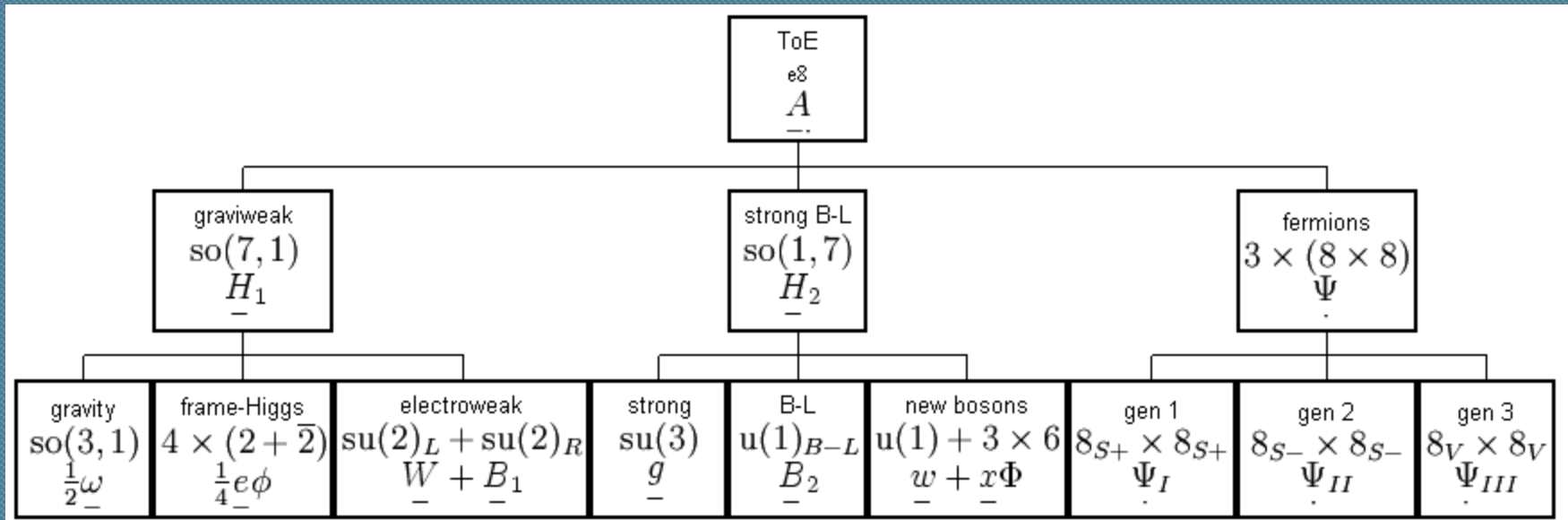


# An Exceptionally Simple Theory of Everything

Notes by Chris Lomont, [LOMONT.org](http://LOMONT.org), April 2010



# Abstract to explain

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- Garrett Lisi's paper : "An Exceptionally Simple Theory of Everything"
  - <http://arxiv.org/abs/0711.0770>
- Abstract: (I want to motivate what the blue words mean)
- All **fields** of the **standard model** and **gravity** are unified as an **E8 principal bundle connection**. A non-compact real form of the **E8 Lie algebra** has **G2** and **F4 subalgebras** which break down to **strong su(3)**, **electroweak su(2) x u(1)**, **gravitational so(3,1)**, the **frame-Higgs**, and three generations of **fermions** related by triality. The interactions and dynamics of these **1-form** and **Grassmann** valued parts of an **E8 superconnection** are described by the **curvature** and **action** over a **four dimensional base manifold**.

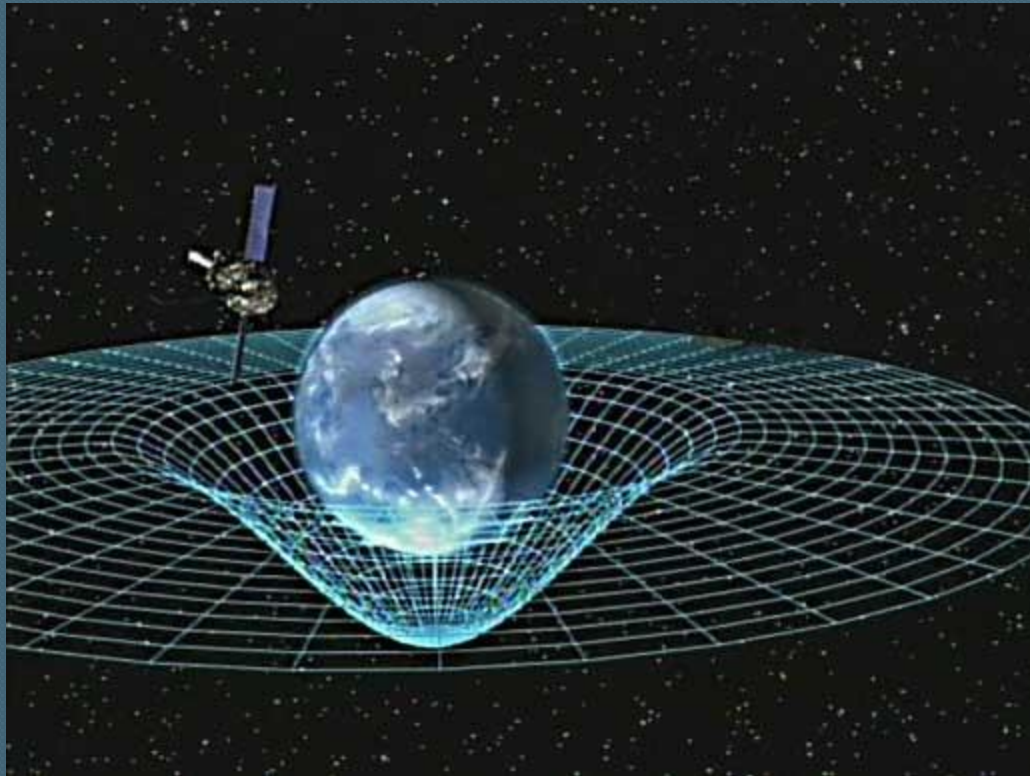
# Introduction

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- We want to unify the main theories of how the universe works into a comprehensive framework, a “Theory of Everything”
- Currently, all known physical phenomena are covered by two remaining theories, relativity and quantum mechanics
- Problem: general relativity and quantum mechanics are at odds with each other

# Spacetime

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# The Concept of Space (and time)

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- Introductory physics uses Euclidean space (x,y,z)
- Distance is coordinate system invariant
  - $d^2 = x^2 + y^2 + z^2$
  - This defines a “metric” which measures properties about space.
- Time is invariant (same everywhere)
  - Galilean transformations
- Coordinates
  - Think of coordinates x, y, z, as “functions” on the points of space
  - Commutative:  $xy = yx$
  - Continuous

# Special Relativity

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- 3D plus time =  $(x,y,z,t)$  = **4D “events”**
  - Distance no longer invariant
  - Time no longer invariant
  - Space-time *interval* is invariant
$$d^2 = t^2 - x^2 - y^2 - z^2$$
    - Follows quickly from speed of light same for all observers.
  - Strange – can have nonzero vectors with 0 length
- Extensively tested, replaces Euclidean geometry as underlying reality

# Extending Space(time)

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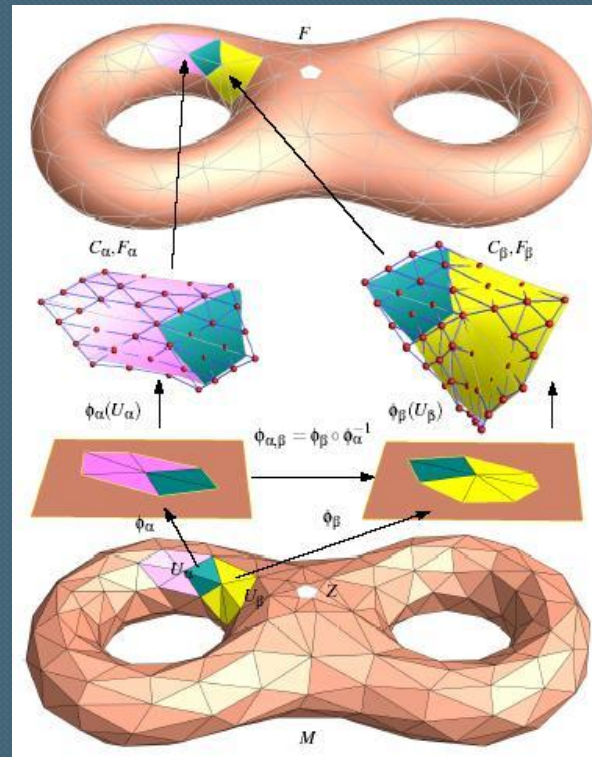
- Replace the absolute coordinate systems
- Space and time are “equal” and intertwined
- Gravity forces idea of *curvature* of spacetime
- Globally, space can be curved
  - Example: earth seems “flat” locally, globally is spherical
  - Example: torus
  - Metric based on “geodesics”
- General Relativity  $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ 
  - Matter and curvature are connected at a deep level

# Predictions

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- General (and Special) Relativity predictions
  - Possibility of non-static universe
  - Black Holes
  - Light bending
  - Gravitational Waves
  - Time Dilation
  - Length Contraction
  - Mercury Perihelion Shift
  - $E=mc^2$
  - Magnetic force is merely relativistic interpretation of electric force, hence the “electromagnetic force”.

# Manifolds – Abstractions of Space



# Concept of a Manifold

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- Locally Euclidean patches “glued” with differentiable functions
- Need these functions to do calculus in order to do physics
  - Differential equations describe motion in a patch.
  - Across patches things are harder.....
  - Physics “tensors” – think 3 axes of E define field at a point
  - Leads to concept of a connection
- For general relativity, need these manifolds to have metric with correct “signature” (1,1,1,-1) called (3,1)

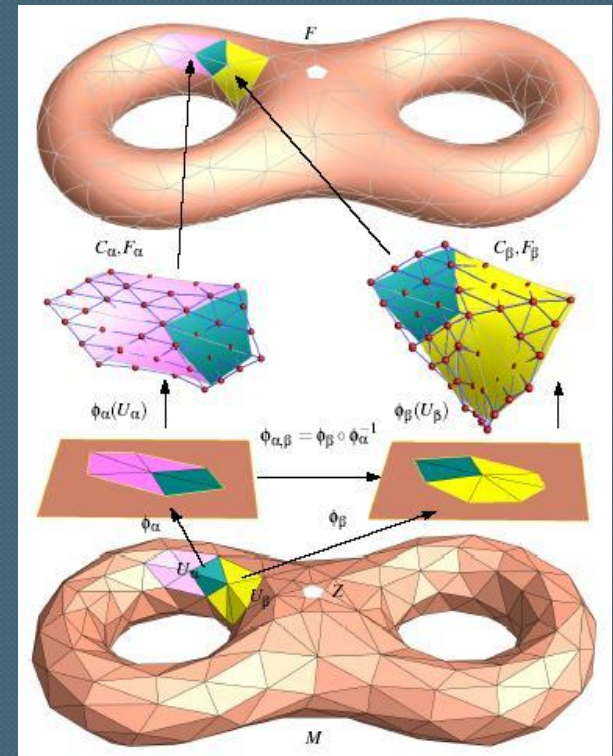
$$d^2 = t^2 - x^2 - y^2 - z^2$$

# Manifold examples

- Examples: coordinates on

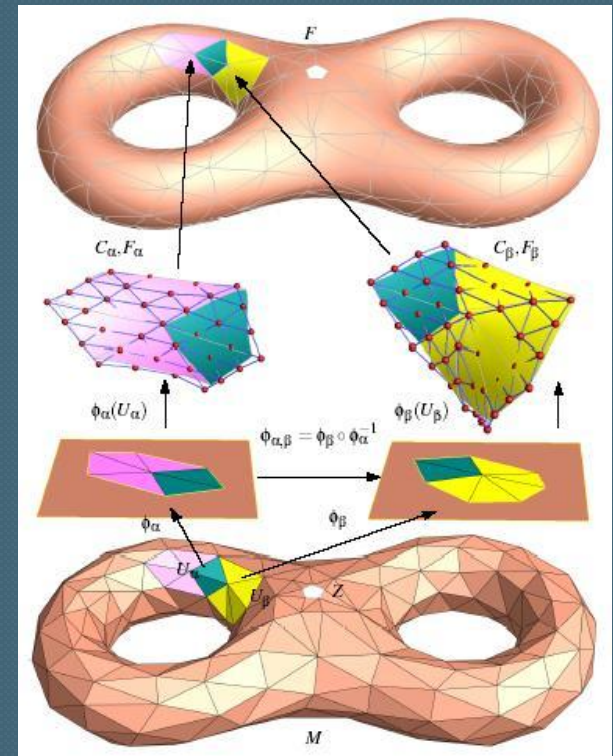
- Sphere
- Torus
- Klein bottle

- Euclidean only true “locally”



# Connections

- Differentiable gluing functions not enough to allow physics to be done
- Need another structure – the connection – which tells how “tensors” move from patch to patch



# Spacetime

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- The resulting 4D framework unifying space and time manifold is called spacetime.
- Spacetime is a manifold, intrinsic curvature based on matter/energy density at each point.
- Any theory of physics must agree on common domain (relativity and QM at overlapping scale)
- To be consistent with relativity any theory of everything must use spacetime as the underlying manifold, not traditional  $(x,y,z)$  Euclidean space.

# Symmetries I

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# Symmetry + Physics

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## ○ Rough plan

- Any complete physical theory must respect the symmetries observed in nature.
- Symmetries are classified mathematically.
- Only structures containing observed symmetries can underlie final theory.
- Vastly restricts possible theories of physics.

# Example

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- Example of how abstract math gives physical theory:
- Distance between points:
  - Relies on coordinate choice
  - Want “distance” to be rotationally invariant, coordinate invariant
  - Pick one coordinate system
    - $f(x_1, y_1, x_2, y_2) = \text{Taylor series} \dots$
  - Need rotations to return consistent answer – fixes the correct answer.

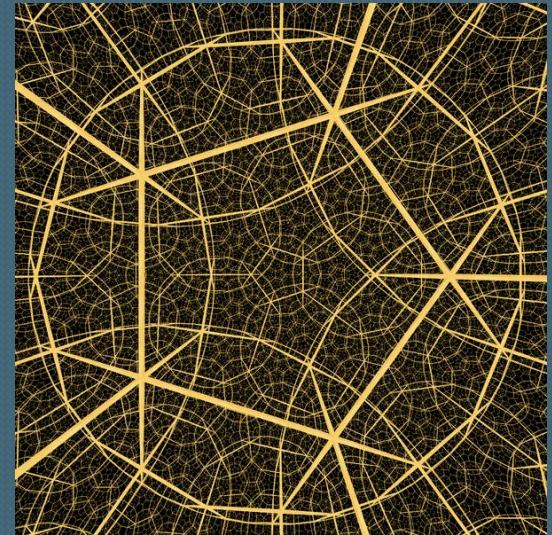
# Noether's Theorem

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- For each conserved quantity, there is a symmetry of the universe, and vice versa.
  - Same dimension, 1D = 1D, 3D=3D
  - Time invariance = conservation of energy
  - Space invariance = conservation of linear momentum
  - Rotational invariance = conservation of angular momentum
- Space-time symmetries lead to mass-energy conservation.
- The language to study symmetries carefully is “group theory”

# Group Theory

- Groups are often the language to study symmetries
- A group is.....
  - Set with operation... (closed, identity, inverse, associative)
- Examples:
  - Integers :  $(\mathbb{Z}, +)$
  - Nonzero complex numbers  $(\mathbb{C}^*, \cdot)$
  - Dihedral groups :  $D_n$
  - Permutation :  $S_n$
  - Rotations of a circle  $O(1)$ , and  $SO(1)$
  - 3 space rotations :  $O(3)$
  - 3 space oriented rotations  $SO(3)$
  - **Lorentz group  $O(3,1)$**



# Simple Groups

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- Groups can have subgroups
  - Examples:
    - Even integers in integers with +
    - Rotations of square in rotations of 8-gon
    - Rational angle rotations of circle in all rotations of circle
- Normal subgroups
  - Used to decompose and study groups the same way primes are used to decompose and study integers
  - Definition somewhat technical...
- Simple groups: only has trivial normal subgroups
  - Groups studied by simple subgroups
  - ***Used to build all other groups like primes used to build integers.***

# Lie Groups

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- Lie group is both a group and a manifold
  - Circle – manifold is circle shape, each point is a group element for a rotation
  - Sphere - similar
  - Invertible matrices – clearly a group, manifold?
  - Translations in the plane – clearly a group, manifold?
- Some Lie Groups
  - $O(n)$  - orthogonal groups (rotations)
  - $U(n)$  – unitary groups (matrices)
    - $SU(n)$  – special unitary
- Allows analysis of *continuous* symmetries, needed in physics.

# Lie group classification

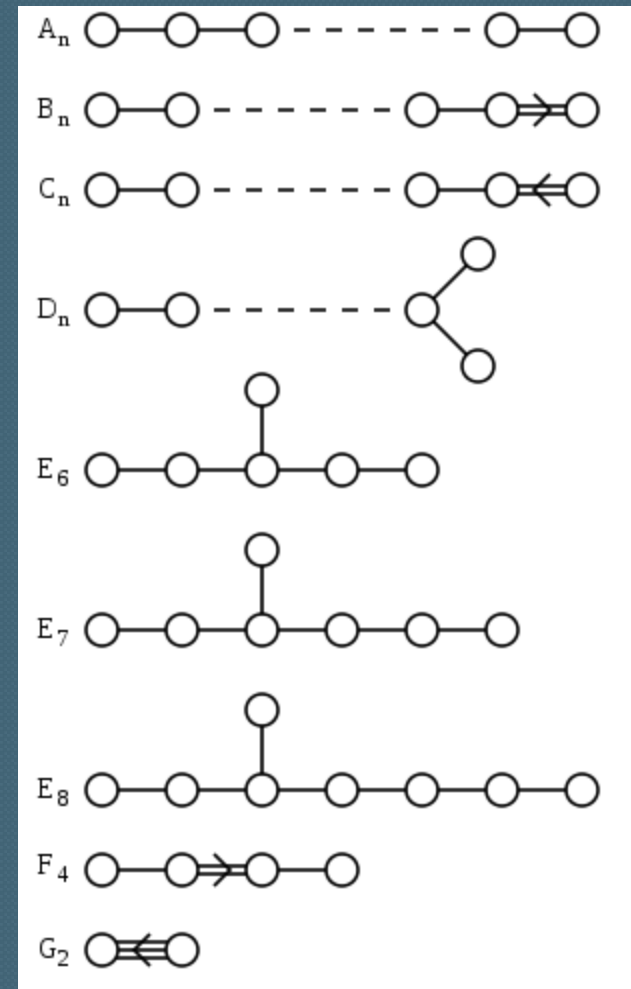
- Lie Group  $\leftrightarrow$  Lie Algebra

- 4 infinite series

- $A_r$  corresponds to the [special unitary group,  \$SU\(r+1\)\$](#) .
- $B_r$  corresponds to the [special orthogonal group,  \$SO\(2r+1\)\$](#) .
- $C_r$  corresponds to the [symplectic group,  \$Sp\(2r\)\$](#) .
- $D_r$  corresponds to the [special orthogonal group,  \$SO\(2r\)\$](#) .

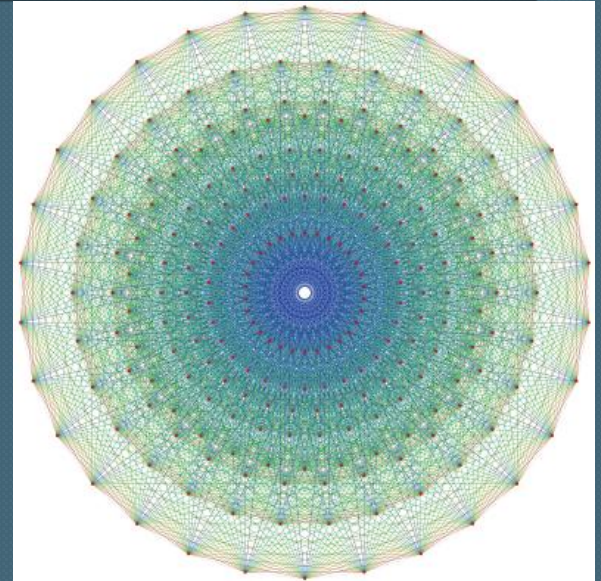
- 5 Exceptional cases

- $G_2$  can be described as the [automorphism group](#) of the [octonion algebra](#)
- $F_4$  has rank 4 and dimension 52. The compact form is simply connected and its [outer automorphism group](#) is the [trivial group](#). Its [fundamental representation](#) is 26-dimensional.
- $E_6$  has rank 6 and dimension 78
- $E_7$  has rank 7 and dimension 133
- $E_8$  has [rank](#) 8 and dimension 248 (as a [manifold](#))



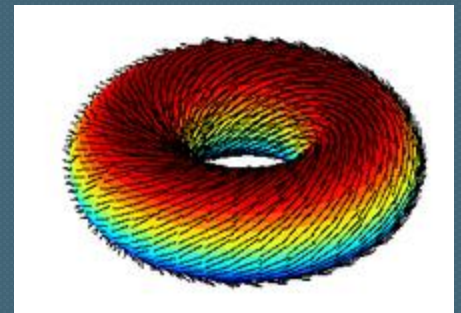
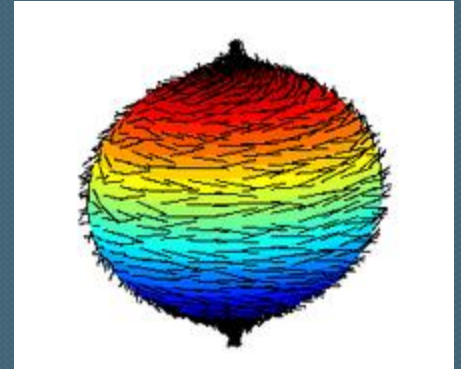
# E8

- Largest Exceptional Group
- Recent research into the group has been acclaimed as the scaling of a mathematical Mount Everest.
- German mathematician Wilhelm Killing first formulated E8 nearly 120 years ago, but it was only in January 2007 that a team of mathematicians completed a detailed map of E8's inner workings.
- The “map,” a table of integers with more than 450,000 rows and columns, required 77 hours of supercomputer time to compute and occupied 60 gigabytes of disk space. For comparison, all the genetic information in a person's cells fills three gigabytes.
- The program to compute the map took more than three years to write.



# Example

- How math bounds reality:
  - Can the wind on each point of the earth be blowing at the same time?
  - Equivalent to drawing a vector field on a sphere.
  - Mathematically, no nonzero vector field can exist on a sphere.
  - So wind cannot blow at each point.
- Note the math question needs no modeling or mention of physical systems.
- Vector field can be smoothed on a torus



# Example

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## ○ Maxwell equations

- Merged many previous formulas under simple equations
- Had a  $U(1)$  symmetry
- Equations predicted that light was a self sustaining electromagnetic wave.
- Quantizing the field leads to the photon.
- Led to gauge theories.

# Physics Symmetries

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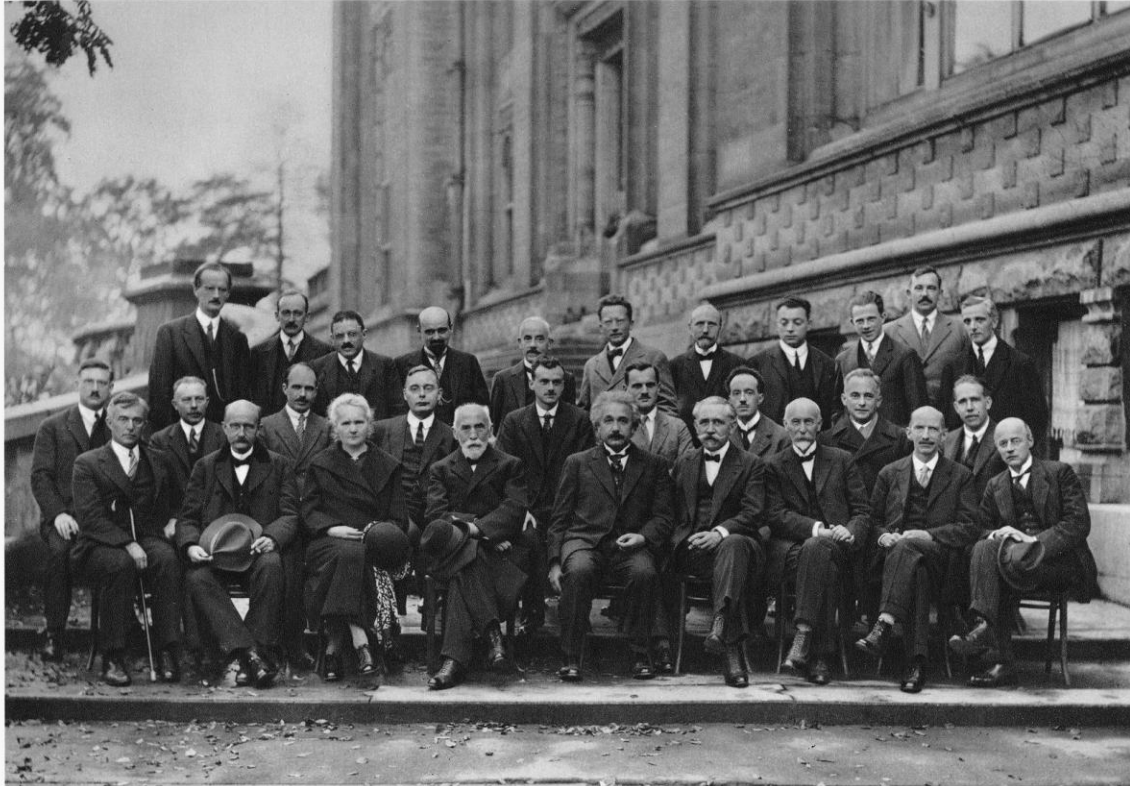
- $SO(3,1)$  - relativity
- $U(1)$  – electromagnetic
- $SU(2)$  - weak
- $SU(3)$  – strong

# Known Symmetries

- Internal symmetry
- (independent of space-time coordinates)

U(1) gauge transformation	electric charge
U(1) gauge transformation	lepton generation number
U(1) gauge transformation	hypercharge
U(1) <sub>Y</sub> gauge transformation	weak hypercharge
U(2) [U(1)xSU(2)]	electroweak force
SU(2) gauge transformation	isospin
SU(2) <sub>L</sub> gauge transformation	weak isospin
PxSU(2)	G-parity
SU(3) "winding number"	baryon number
SU(3) gauge transformation	quark color
SU(3) (approximate)	quark flavor
S((U2)xU(3)) [U(1)xSU(2)xSU(3)]	Standard Model

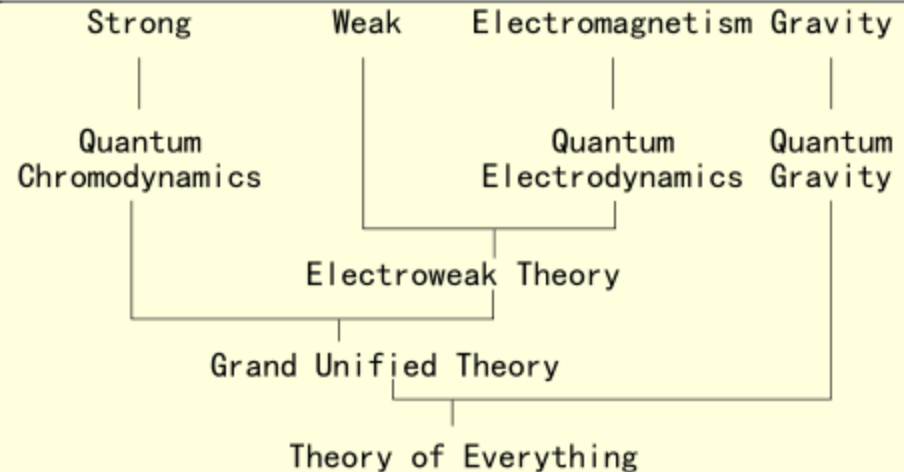
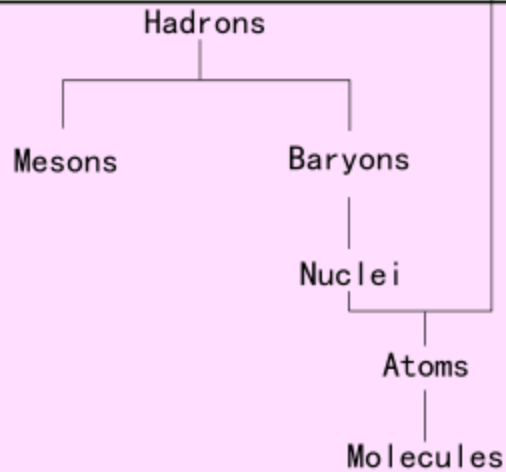
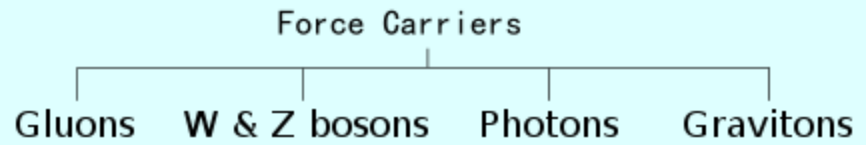
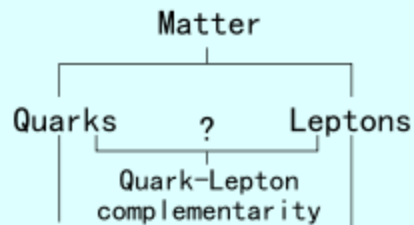
# Quantum Mechanics



A. PICCARD    E. HENRIOT    P. EHRENFEST    Ed. HERZEN    Th. DE DONDER    E. SCHRÖDINGER    E. VERSCHAFFELT    W. PAULI    W. HEISENBERG    R.H. FOWLER    L. BRILLOUIN  
P. DEBYE    M. KNUDSEN    W.L. BRAGG    H.A. KRAMERS    P.A.M. DIRAC    A.H. COMPTON    L. de BROGLIE    M. BORN    N. BOHR  
I. LANGMUIR    M. PLANCK    Mme CURIE    H.A. LORENTZ    A. EINSTEIN    P. LANGEVIN    C.E. GUYE    C.T.R. WILSON    O.W. RICHARDSON  
Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL

# Particle Physics Overview

## *Elementary Particles*



*Composite Particles*

*Forces*

# Changes from classical

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- Energy of black body photons is “quantized”
- Particles have both wavelike and particle-like behavior
- Superposition of states
- Operators
- Hermitian = observable = real eigenvalues

# Quantum Theory

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- Coordinates “non-commutative”
  - Momentum and position non-commutative
    - $mp \neq pm$  so breaks usual space and manifold concepts
  - Comes from wave duality nature of particles
  - Heisenberg Uncertainty
  - Consider “space” of (position, momentum) =  $(p_x, p_y, p_z, m_x, m_y, m_z)$  representing measurements

# Quantum State

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- State is a vector  $|a\rangle$  in a “state space”
- Time evolution is an operator  $H$
- Position is an operator
- Momentum is an operator
- $H$  is the Hamiltonian, changes based on problem
- $E$  is scalar energy

$$H |a\rangle = E_a |a\rangle$$

# Observables

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- Canonical example  $H |a\rangle = E_a |a\rangle$
- The state is a vector
- Operators (think matrices) act on the state
- Eigenvalues correspond to observable measurements
  - Forces eigenvalues to be real (not complex)

# Quantum Numbers

- Describe values of conserved quantities in the dynamics of the quantum system.
- Weird part in QM versus classical is these values are quantized (discrete).
- Electron in an atom only needs 4 numbers:

Name	Symbol, range	Orbital meaning
Principal quantum number	$n=1,2,3,\dots$	Shell
Angular momentum	$0 \leq L \leq n-1$	Subshell
Magnetic quantum number	$-L \leq m \leq L$	Subshell orientation
Spin projection	$-1/2$ or $1/2$ .	Spin

# Spin

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- Not spin like a ball
- It is an intrinsic value, like charge or mass
  - Just a number
- Mathematics of how it works is similar to the mathematics of classical spin, so physicists called it “spin”

# Schrodinger Equation

- The Hamiltonian gives time evolution of the state  $|\psi(t)\rangle$

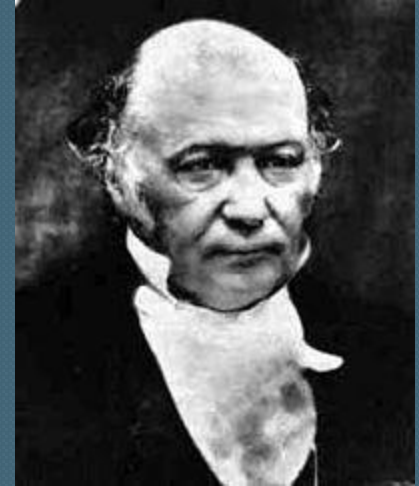
$$H |\psi(t)\rangle = i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle$$



# Hamiltonian

- What is the Hamiltonian?
- Hamilton's equations provide a new and equivalent way of looking at classical mechanics
- $H$  is Hamiltonian operator,
  - $T$  = kinetic,  $V$  = potential.
  - $q$  is position,  $p$  is momentum

$$H = T + V$$



$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}$$
$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial q}$$

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}$$
$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial q}$$

# Hamiltonian Example

○ Kinetic particle mass  $m$   $T = \frac{1}{2} m v^2 = \frac{1}{2} \frac{p^2}{m}, p = m v$

○ Potential Center mass  $M$   $V = -\frac{GMm}{q}$

$$H = T + V = \frac{1}{2} \frac{p^2}{m} - \frac{GMm}{q}$$

$$\dot{p} = \frac{\partial p}{\partial t} = -\frac{\partial H}{\partial q} = -\frac{GMm}{q^2}$$

$$\dot{q} = \frac{\partial q}{\partial t} = \frac{\partial H}{\partial p} = \frac{1}{m} p$$

$$\frac{\partial^2 q}{\partial t^2} = \frac{1}{m} \frac{\partial p}{\partial t} = \frac{1}{m} \left( -\frac{GMm}{q^2} \right) = -\frac{GM}{q^2}$$

$$m a = m \frac{\partial^2 q}{\partial t^2} = -\frac{GMm}{q^2} = F$$

# Lagrangian

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- Equivalent to the Hamiltonian is the Lagrangian  $L=T-V$
- Describes motion of a system
- $L$  is a function that summarizes the dynamics of the system
- Current physics often is specified by giving a Lagrangian (for technical reasons)



# Lagrangian II

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- The Lagrangian must satisfy all known symmetries
  - Restricts possible Lagrangians
  - Describes all observed quantum phenomena
  - Used to describe new theories of physics

# Action

- The integral of the Lagrangian is the Action

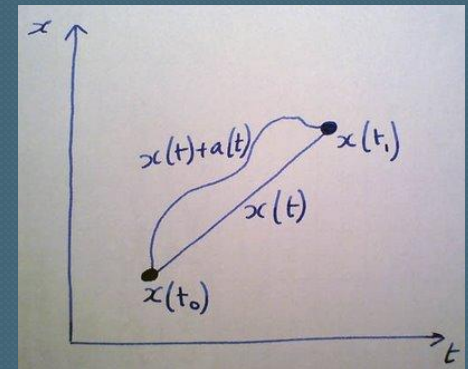
$$S = \int L dt$$

- Classical mechanics

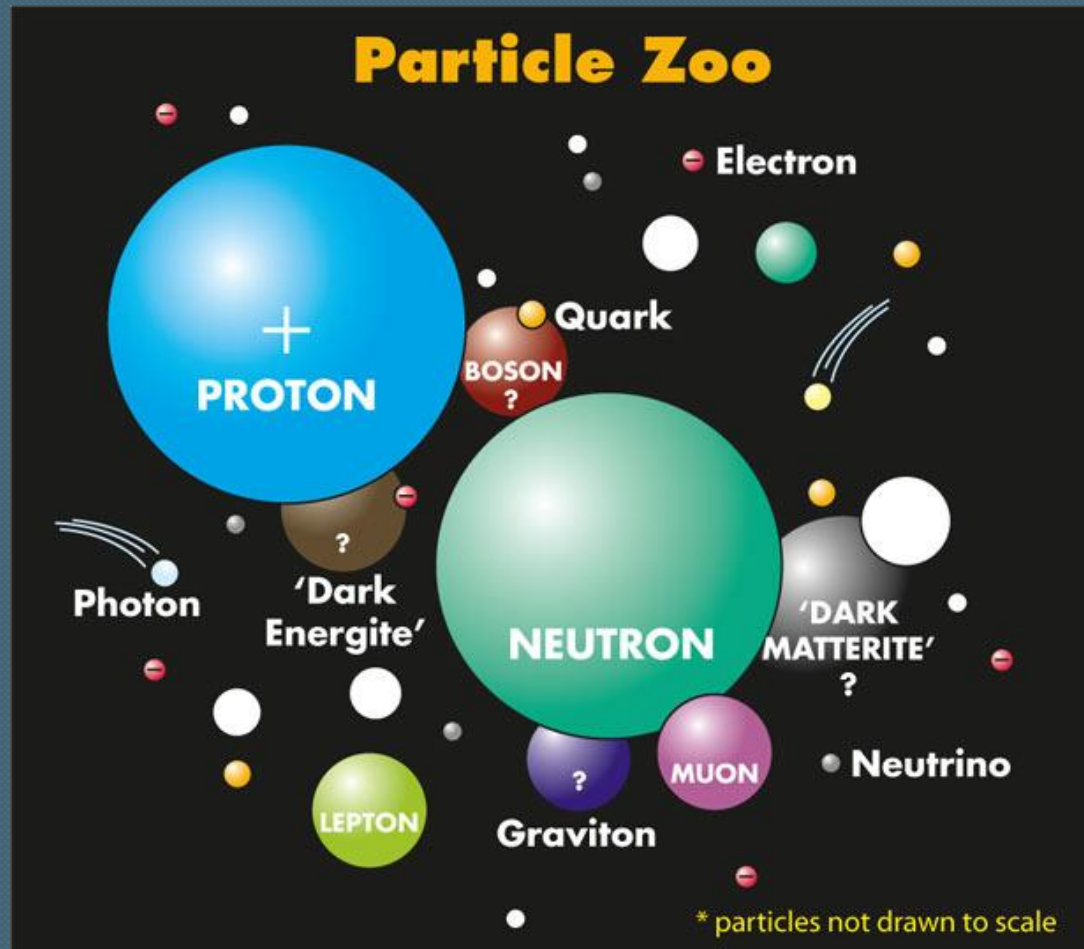
- The path taken by a physical system minimizes the action.

- The principle of least action.

- Specifying an action (or Lagrangian) completely describes a physical system.
- Extends to quantum mechanics
- Basis of the Feynman path integral.
- Provides a basis for the development of string theory.



# Particle Zoo



# Standard Model of FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

## FERMIONS

**matter constituents**  
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c <sup>2</sup>	Electric charge	Flavor	Approx. Mass GeV/c <sup>2</sup>	Electric charge
$\nu_e$ electron neutrino	$<1 \times 10^{-8}$	0	<b>u</b> up	0.003	2/3
<b>e</b> electron	0.000511	-1	<b>d</b> down	0.006	-1/3
$\nu_\mu$ muon neutrino	$<0.0002$	0	<b>C</b> charm	1.3	2/3
<b><math>\mu</math></b> muon	0.106	-1	<b>S</b> strange	0.1	-1/3
$\nu_\tau$ tau neutrino	$<0.02$	0	<b>t</b> top	175	2/3
<b><math>\tau</math></b> tau	1.7771	-1	<b>b</b> bottom	4.3	-1/3

**Spin** is the intrinsic angular momentum of particles. Spin is given in units of  $\hbar$  which is the quantum unit of angular momentum, where  $\hbar = h/2\pi = 6.58 \times 10^{-25} \text{ GeV s} = 1.05 \times 10^{-34} \text{ J s}$ .

**Electric charges** are given in units of the proton's charge. In SI units the electric charge of the proton is  $1.60 \times 10^{-19}$  coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in  $\text{GeV}/c^2$  (remember  $E = mc^2$ ), where  $1 \text{ GeV} = 10^9 \text{ eV} = 1.60 \times 10^{-10} \text{ joule}$ . The mass of the proton is  $0.938 \text{ GeV}/c^2 = 1.67 \times 10^{-27} \text{ kg}$ .

## BOSONS

**force carriers**  
spin = 0, 1, 2, ...

Unified Electroweak spin = 1			Strong (color) spin = 1		
Name	Mass GeV/c <sup>2</sup>	Electric charge	Name	Mass GeV/c <sup>2</sup>	Electric charge
$\gamma$ photon	0	0	<b>g</b> gluon	0	0
<b>W<sup>-</sup></b>	80.4	-1			
<b>W<sup>+</sup></b>	80.4	+1			
<b>Z<sup>0</sup></b>	91.187	0			

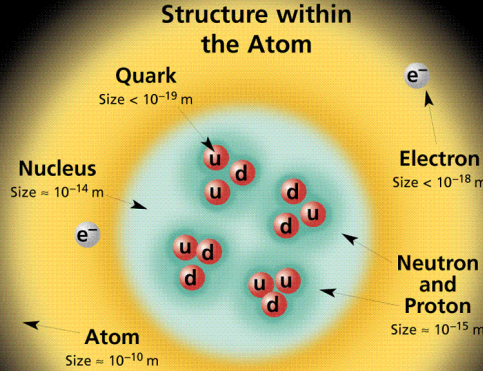
**Color Charge**  
Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and **W** and **Z** bosons have no strong interactions and hence no color charge.

### Quarks Confined in Mesons and Baryons

One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons**  $q\bar{q}$  and **baryons**  $qqq$ .

### Residual Strong Interaction

The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.



If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

## PROPERTIES OF THE INTERACTIONS

Baryons $qqq$ and Antibaryons $\bar{q}\bar{q}\bar{q}$					
Baryons are fermionic hadrons. There are about 120 types of baryons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c <sup>2</sup>	Spin
<b>p</b>	proton	<b>uud</b>	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	-1	0.938	1/2
<b>n</b>	neutron	<b>udd</b>	0	0.940	1/2
$\Lambda$	lambda	<b>uds</b>	0	1.116	1/2
$\Omega^-$	omega	<b>sss</b>	-1	1.672	3/2

Property	Gravitational	Weak (Electroweak)	Electromagnetic	Strong	
	Mass - Energy	Flavor	Electric Charge	Fundamental	Residual
<b>Acts on:</b>	<b>Mass - Energy</b>	<b>Flavor</b>	<b>Electric Charge</b>	<b>Color Charge</b>	See Residual Strong Interaction Note
<b>Particles experiencing:</b>	<b>All</b>	<b>Quarks, Leptons</b>	<b>Electrically charged</b>	<b>Quarks, Gluons</b>	<b>Hadrons</b>
<b>Particles mediating:</b>	<b>Graviton</b> (not yet observed)	<b>W<sup>+</sup> W<sup>-</sup> Z<sup>0</sup></b>	<b><math>\gamma</math></b>	<b>Gluons</b>	<b>Mesons</b>
<b>Strength relative to electromag for two u quarks at:</b>					
for two u quarks at:	$10^{-18} \text{ m}$	0.8	1	25	Not applicable to quarks
	$3 \times 10^{-17} \text{ m}$	$10^{-4}$	1	60	
	for two protons in nucleus	$10^{-36}$	$10^{-7}$	1	

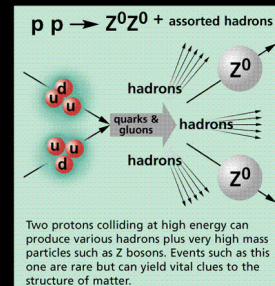
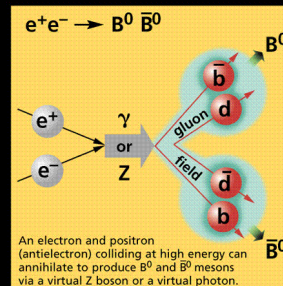
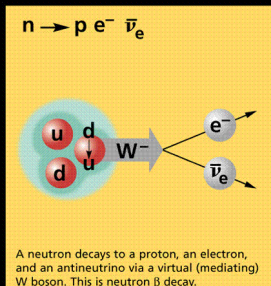
Mesons $q\bar{q}$					
Mesons are bosonic hadrons. There are about 140 types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c <sup>2</sup>	Spin
$\pi^+$	pion	<b>u<math>\bar{d}</math></b>	+1	0.140	0
<b>K<sup>-</sup></b>	kaon	<b>s<math>\bar{u}</math></b>	-1	0.494	0
<b><math>\rho^+</math></b>	rho	<b>u<math>\bar{d}</math></b>	+1	0.770	1
<b>B<sup>0</sup></b>	B-zero	<b>d<math>\bar{b}</math></b>	0	5.279	0
$\eta_c$	eta-c	<b>c<math>\bar{c}</math></b>	0	2.980	0

### Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or - charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g.,  $Z^0$ ,  $\gamma$ , and  $\eta_c = c\bar{c}$ , but not  $K^0 = d\bar{s}$ ) are their own antiparticles.

### Figures

These diagrams are an artist's conception of physical processes. They are **not** exact and have **no** meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.



### The Particle Adventure

Visit the award-winning web feature *The Particle Adventure* at <http://ParticleAdventure.org>

This chart has been made possible by the generous support of:

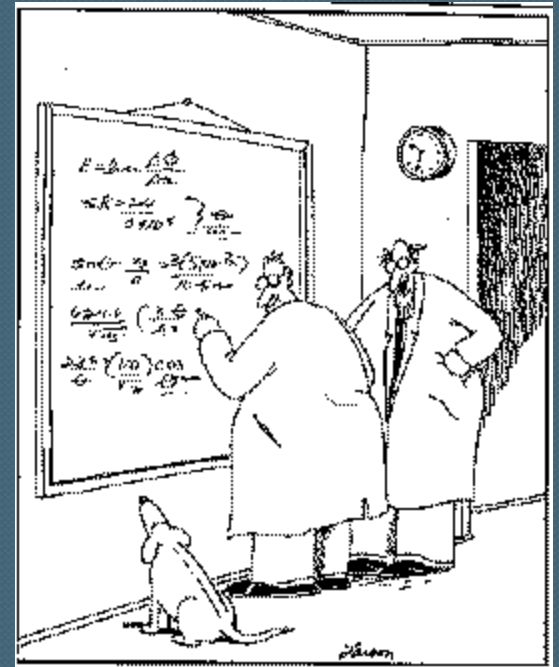
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# Quantum Field Theory

- Abstract quantum mechanics one level.....
- Marries quantum mechanics with special relativity!
- No more particles
  - Now all is “fields”



They act so cute when they try to understand Quantum Field Theory.

# QFT

---

- ◉ QM often operates in the domain of special relativity (high speed, energy, etc).
- ◉ Thus relativistic QM needed
- ◉ Leads to QFT

# Fields

---

- At each point of space(time) there is some measurable quantity, direction, etc.
  - Called a “field”
  - Electromagnetic field (vector)
  - Gravitational field (vector)
- Other fields from particle physics:
  - weak forces
  - quark color

# Field Symmetries

---

- Fields often classified by behavior under transformations of spacetime
  - **Scalar** fields: attached a single number to each point of spacetime
    - Example: temperature
    - Invariant under transformations of space.
  - **Vector** fields : attach a vector to each point of spacetime
    - Example: magnetic field has direction and magnitude
    - Components transform between themselves as usual under rotations in space.
  - **Tensor** fields : attach a tensor to each point of space
    - Example: stress tensor in a crystal
    - Components transform between themselves as usual under rotations in space.
  - **Spinor** fields : attach a spinor to each point of space.
    - Useful in quantum field theory.

# Tensors

- Extend the notion of scalars, vectors, matrices
- K-dimensional tensor is a k-dimensional array of values
  - Vector is 1-tensor
  - Matrix is 2-tensor



# Electromagnetic Tensor

---

- Electromagnetic Tensor
- Combines all of Maxwell's equations in compact, mathematically concise form

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

- All of classical electromagnetism can be derived from the action of the simple Lagrangian

$$\mathcal{L} = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu}$$

# Spinors

---

- Rotation of a vector in space loses some of the detail of movement.
  - Spinor keeps track of this more nuanced “rotation”
- Used to study intrinsic quantum properties
  - Angular momentum of electron
- The real spinors in three dimensions are the quaternions, useful for rotations
  - Used in computer games
  - Used in CAD systems
  - Used for natural feeling mouse control of 3D

# Internal Field Symmetries

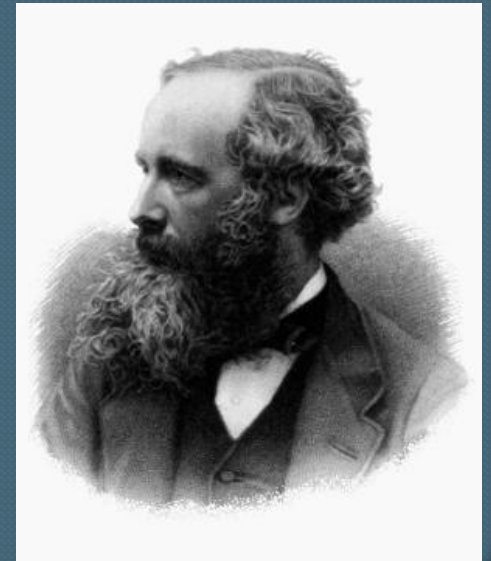
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- Fields may have **internal symmetries** in addition to space-time symmetries.
  - For example, in many situations one needs fields which are a list of space-time scalars:  $(\phi_1, \phi_2 \dots \phi_N)$ . For example, in weather prediction these may be temperature, pressure, humidity, etc.
  - In particle physics
    - color symmetry of the interaction of quarks strong interaction
    - isospin or flavor symmetry.
- If there is a symmetry of the problem, not involving space-time, under which these components transform into each other, then this set of symmetries is called an **internal symmetry**.

# Classical Fields

---

- Classical: electric, magnetic, gravitational
- Dynamics usually specified by a Lagrangian density using the action principle
- Faraday realized electric and magnetic fields are not only fields of force which dictate the motion of particles, but also have an **independent physical reality** because they carry energy.
- James Clerk Maxwell created the first unified field theory by merging the electric and magnetic fields into the electromagnetic field.



# Field Unification

- End of the 19th century, the electromagnetic field was understood as a collection of two vector fields in space.
  - Currently viewed as single antisymmetric 2nd-rank tensor field in space-time.

$$\mathcal{L} = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu}$$

- Einstein's general relativity is another example of a field theory.
  - Here the principal field is the metric tensor, a symmetric 2nd-rank tensor field in space-time.
  - Describes how mass and energy (stress-energy tensor) are related to the curvature of space-time (as represented in the Einstein tensor).

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

# Stress Energy Tensor

---

- Describes density and flux of energy and momentum in space-time
- Generalizes stress tensor of Newtonian mechanics
- Source of the gravitational field
  - Just as mass is the source in Newtonian

# Curvature

---

- $R_{uv}$  is Ricci curvature tensor
  - Intrinsic to a manifold
  - Amount by which the volume element in a curved manifold deviates from that of standard Euclidean space
- $R$  is scalar curvature
  - Derived from Ricci
- $g_{uv}$  is metric tensor
  - Defines distances, angles, and similar measurements on a manifold
- $G_{uv}$  is Einstein field tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$$

# Quantum Fields

- Classical field can be recast as a QFT
  - Electromagnetic field gives Quantum Electrodynamics (photon/electron interactions)
  - QED most successful scientific theory – matches experiment to 10 parts in a billion

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- Other fundamental quantum field theories are
  - quantum chromodynamics – strong force
  - electroweak theory – weak force
- Gravity quantization predicts (unseen) graviton.

# Known Forces

Force	Theory	Mediators	Relative strength	Range (m)
Gravitation	General Relativity (GR)	Gravitons (unseen)	1	Infinite
Weak	Electroweak	W and Z bosons	$10^{25}$	$10^{-18}$
Electromagnetic	Quantum Electrodynamics (QED)	Photons	$10^{36}$	Infinite
Strong	Quantum Chromodynamics (QCD)	gluons	$10^{38}$	$10^{-15}$

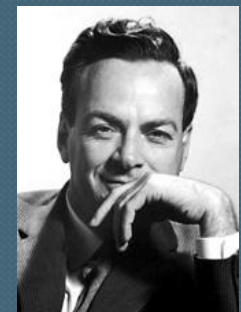
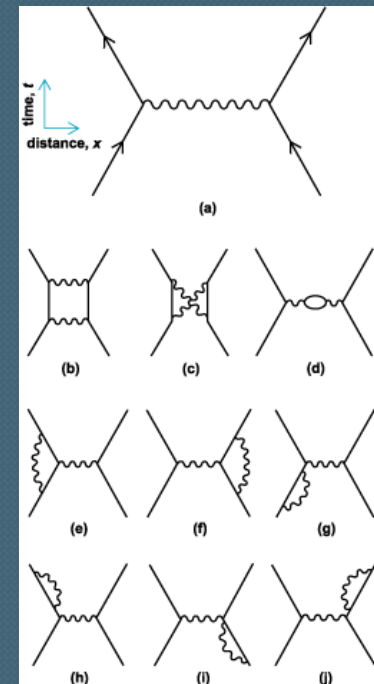
# Electromagnetic Unification

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- In 1861, James Clerk Maxwell reduced all of the current knowledge into a linked set of differential equations with 20 equations in 20 variables.
- U(1) symmetry
- Predicts light is electromagnetic phenomena 1862.

# Quantum Electrodynamics

- Describes how light and matter interact.
- Three basic QED actions
  - A photon goes from one place and time to another place and time.
  - An electron goes from one place and time to another place and time.
  - An electron emits or absorbs a photon at a certain place and time.
- Most accurate scientific theory
- U(1) symmetry



$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

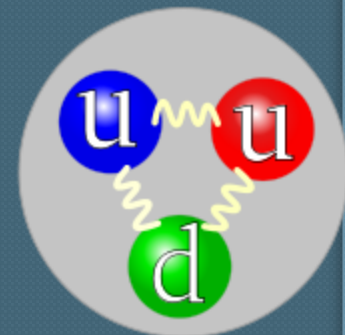
# Electroweak Unification

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- Unifies electromagnetic and weak forces
  - in 1968
- $SU(2) \times U(1)$  symmetry
- Salam, Glashow, Weinberg 1979 Nobel
- Photon for EM
- $W_{\pm}$  and  $Z$  mediate weak force
  - Discovered in 1983 at CERN

# Quantum Chromodynamics

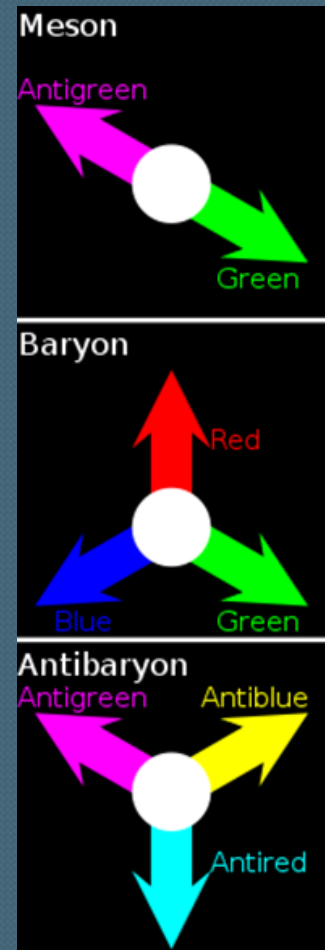
- Developed purely from theory 1964 Gell-Mann and Zweig
  - Predicts 6 quark flavors: up, down, charm, strange, top, bottom
  - Predicts gluons that exchange colors of quarks
  - First quark seen in 1968. Last (top quark) found 1998.
- QCD is QFT** theory of the strong interaction (color force)
  - Describes quarks and gluons interactions
  - Makes up hadrons
  - Includes proton, neutron or pion.
- SU(3) symmetry of color-charged fermions (the quarks).
- Quark **flavors** prompted by the necessity of explaining the properties of hadrons
- The notion of **color** was necessitated by the puzzle of the  $\Delta^{++}$ . (Red, Green, Blue)



Proton =  
u+u+d quark

# Quarks

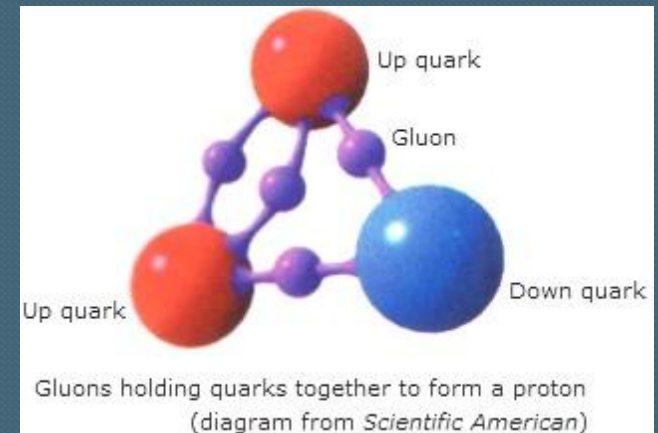
- Massive spin-1/2 fermions
- Have color charge
- Electric charge  $-1/3$  or  $2/3$
- Participate in weak interactions
- Global quantum numbers
  - baryon number, which is  $1/3$  for each quark,
  - hypercharge
  - flavor quantum numbers.
- Every quark has its own antiquark.
  - Opposite charge
- Never found in isolation
- Only elementary particle to experience all 4 fundamental forces.



# Gluons

- Spin-1 bosons
- Carry color charges
- Have no electric charge
- Do not participate in the weak interactions
- Have no flavor.

$$(r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$$



$$\begin{array}{ll} (r\bar{b} + b\bar{r})/\sqrt{2} & -i(r\bar{b} - b\bar{r})/\sqrt{2} \\ (r\bar{g} + g\bar{r})/\sqrt{2} & -i(r\bar{g} - g\bar{r})/\sqrt{2} \\ (b\bar{g} + g\bar{b})/\sqrt{2} & -i(b\bar{g} - g\bar{b})/\sqrt{2} \\ (r\bar{r} - b\bar{b})/\sqrt{2} & (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6} \end{array}$$

# Particle Unification

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- ◉ QED
  - Force and carrier particles
- ◉ QCD
  - Force and carrier particles
- ◉ Quantum Field Theories
- ◉ Current unification: the Standard Model

# Standard Model

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- Current state of classification of elementary particles.
- Describes the strong, weak, and electromagnetic fundamental forces
  - gauge bosons are the gluons
  - $W^-$  and  $W^+$  and Z bosons
  - photons.
- Contains 24 fundamental particles, the constituents of matter.
- Predicts the existence of the Higgs boson (currently discovered).
- The most general Lagrangian dynamics depend on 19 parameters
  - numerical values are established by experiment.
- Symmetry group  $SU(3) \times SU(2) \times U(1)$ .
- To date, all experimental tests of the three forces described by the Standard Model have agreed with its predictions.

### Parameters of the Standard Model

Symbol	Description	Renormalization scheme (point)	Value
$m_e$	Electron mass		511 keV
$m_\mu$	Muon mass		106 MeV
$m_\tau$	Tauon mass		1.78 GeV
$m_u$	Up quark mass	$\mu_{\overline{MS}} = 2 \text{ GeV}$	1.9 MeV
$m_d$	Down quark mass	$\mu_{\overline{MS}} = 2 \text{ GeV}$	4.4 MeV
$m_s$	Strange quark mass	$\mu_{\overline{MS}} = 2 \text{ GeV}$	87 MeV
$m_c$	Charm quark mass	$\mu_{\overline{MS}} = m_c$	1.32 GeV
$m_b$	Bottom quark mass	$\mu_{\overline{MS}} = m_b$	4.24 GeV
$m_t$	Top quark mass	On-shell scheme	172.7 GeV
$\theta_{12}$	CKM 12-mixing angle		13.1°
$\theta_{23}$	CKM 23-mixing angle		2.4°
$\theta_{13}$	CKM 13-mixing angle		0.2°
$\delta$	CKM CP-violating Phase		0.995
$g_1$	U(1) gauge coupling	$\mu_{\overline{MS}} = m_Z$	0.357
$g_2$	SU(2) gauge coupling	$\mu_{\overline{MS}} = m_Z$	0.652
$g_3$	SU(3) gauge coupling	$\mu_{\overline{MS}} = m_Z$	1.221
$\theta_{\text{QCD}}$	QCD vacuum angle		~0
$\mu$	Higgs quadratic coupling		Unknown
$\lambda$	Higgs self-coupling strength		Unknown

# Standard Model

- Electroweak Lagrangian portion

$$\mathcal{L}_{EW} = \sum_{\psi} \bar{\psi} \gamma^{\mu} \left( i \partial_{\mu} - g' \frac{1}{2} Y_W B_{\mu} - g \frac{1}{2} \vec{\tau}_L \vec{W}_{\mu} \right) \psi + \mathcal{L}_{YM}(B_{\mu}) + \mathcal{L}_{YM}(\vec{W}_{\mu}),$$

- Quantum Chromodynamics portion

$$\begin{aligned} \mathcal{L}_{QCD} &= \bar{\psi}_i (i \gamma^{\mu} (D_{\mu})_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \\ &= \bar{\psi}_i (i \gamma^{\mu} \partial_{\mu} - m) \psi_i - g G_{\mu}^a \bar{\psi}_i \gamma^{\mu} T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}, \end{aligned}$$

- Higgs Lagrangian portion

$$\mathcal{L}_H = \varphi^{\dagger} \left( \overleftarrow{\partial}_{\mu} - \frac{i}{2} (g' Y_W B_{\mu} + g \vec{\tau} \vec{W}_{\mu}) \right) \left( \partial_{\mu} + \frac{i}{2} (g' Y_W B_{\mu} + g \vec{\tau} \vec{W}_{\mu}) \right) \varphi - \frac{\lambda^2}{4} (\varphi^{\dagger} \varphi - v^2)^2.$$

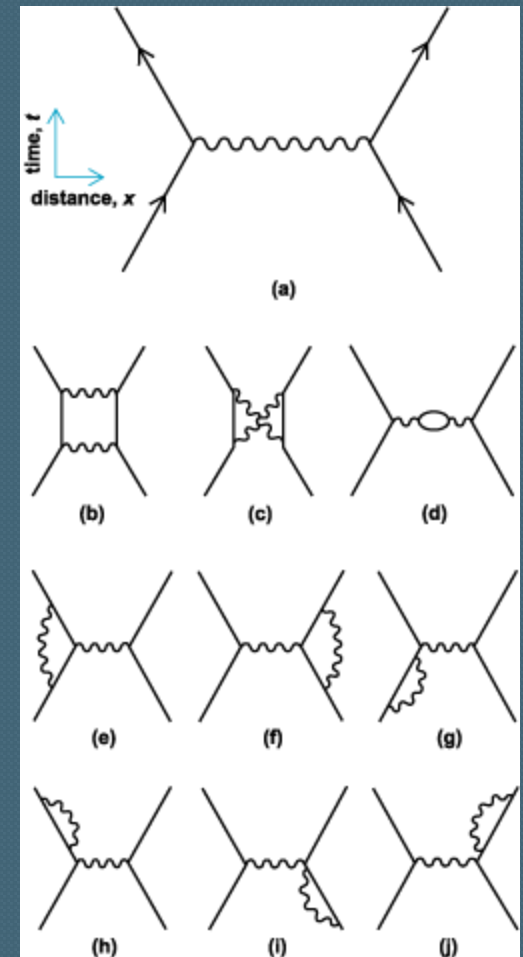
# Standard Model Symmetries

**Symmetries of the Standard Model and Associated Conservation Laws**

Symmetry	Lie Group	Symmetry Type	Conservation Law
Poincaré	Translations $\times$ SO(3,1)	Global symmetry	Energy, Momentum, Angular momentum
Gauge	SU(3) $\times$ SU(2) $\times$ U(1)	Local symmetry	Color charge, Weak isospin, Electric charge, Weak hypercharge
Baryon phase	U(1)	Accidental Global symmetry	Baryon number
Electron phase	U(1)	Accidental Global symmetry	Electron number
Muon phase	U(1)	Accidental Global symmetry	Muon number
Tauon phase	U(1)	Accidental Global symmetry	Tauon number

# Standard Model Computations

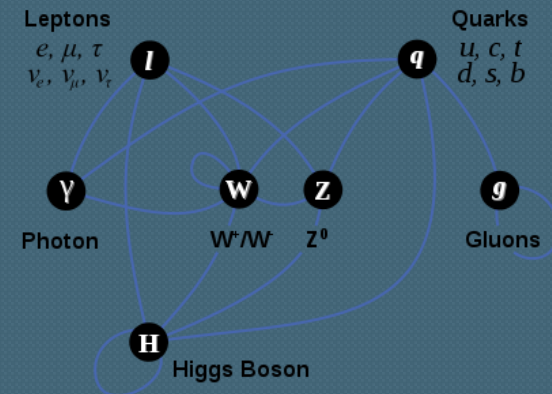
- Used to compute probabilities via Feynman Diagrams
  - Scattering
  - Decay
- Matches all known quantum experiments



# Particles

Quarks	2.4 MeV $\frac{2}{3}$ $\frac{1}{2}$ <b>u</b> up	1.27 GeV $\frac{2}{3}$ $\frac{1}{2}$ <b>c</b> charm	171.2 GeV $\frac{2}{3}$ $\frac{1}{2}$ <b>t</b> top	0 0 1 <b><math>\gamma</math></b> photon
	4.8 MeV $-\frac{1}{3}$ $\frac{1}{2}$ <b>d</b> down	104 MeV $-\frac{1}{3}$ $\frac{1}{2}$ <b>s</b> strange	4.2 GeV $-\frac{1}{3}$ $\frac{1}{2}$ <b>b</b> bottom	0 0 1 <b>g</b> gluon
	<2.2 eV 0 $\frac{1}{2}$ <b><math>\nu_e</math></b> electron neutrino	<0.17 MeV 0 $\frac{1}{2}$ <b><math>\nu_\mu</math></b> muon neutrino	<15.5 MeV 0 $\frac{1}{2}$ <b><math>\nu_\tau</math></b> tau neutrino	91.2 GeV 0 1 <b>Z</b> weak force
Leptons	0.511 MeV $-1$ $\frac{1}{2}$ <b>e</b> electron	105.7 MeV $-1$ $\frac{1}{2}$ <b><math>\mu</math></b> muon	1.777 GeV $-1$ $\frac{1}{2}$ <b><math>\tau</math></b> tau	80.4 GeV $\pm 1$ 1 <b><math>W^\pm</math></b> weak force

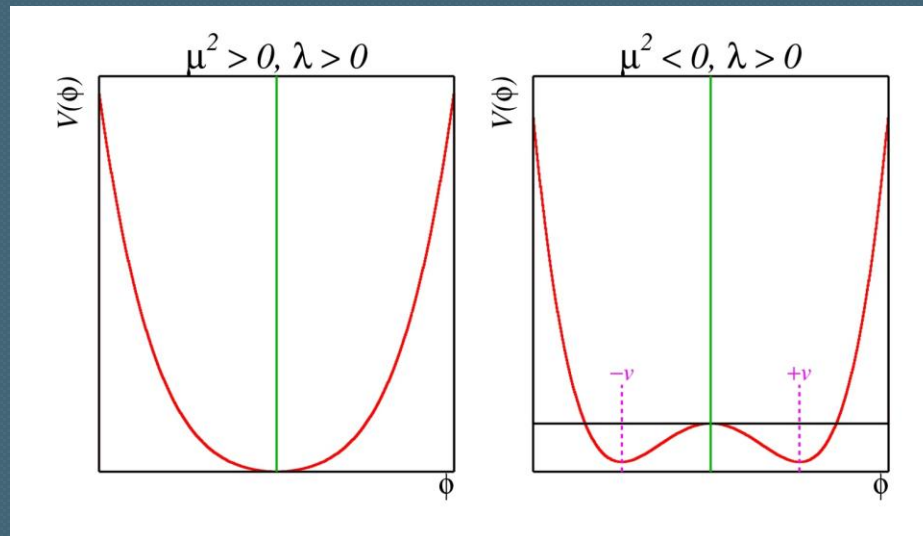
Bosons (Forces)





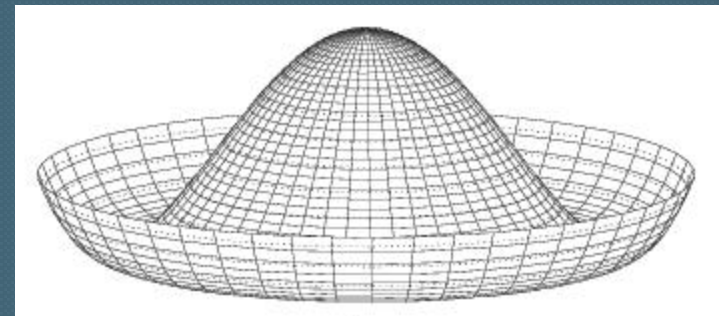
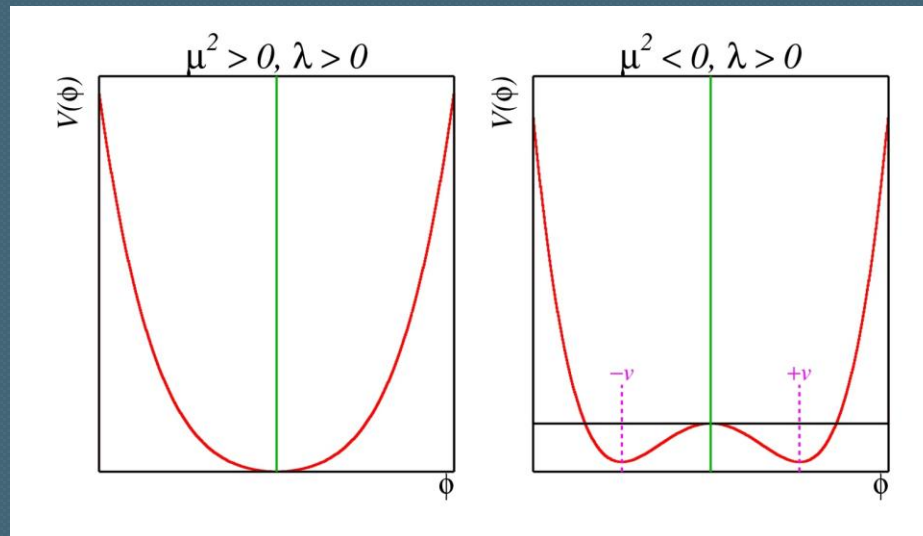
# Higgs Field I

- Start with a *massless* field Lagrangian
  - No mass terms to start
  - Obtain quadratic potential (left)
  - Lowest state corresponds to a boson
  - For EM field, obtain a massless photon



# Higgs Field II

- Start with a *massless* field Lagrangian
  - For more complex symmetries, obtain potential on *right*. Called the Mexican hat. Lowest term at top of hull, wants to roll off under any perturbation.
  - Perturbation, called *symmetry breaking*, mass term pops out.
  - This is the *Higgs mechanism*, gives *Higgs Boson* and mass to interaction.



# Physics Symmetries

---

# Known Symmetries

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- GUT must have  $SU(3) \times SU(2) \times U(1)$  symmetry from Standard Model.
- Must have  $SO(3,1)$  from General Relativity.

# Main GUT Problem

---

- Relativity and these manifolds assume space and time are continuous, smooth
  - Relativity describes large scale of the universe
- Quantum Mechanics details the small parts of the universe
  - Quantum mechanics treats space and time differently, but relativity does not!
- Relativity commutative, QM not

# Math aside - Bundles

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# Bundles

- Line bundle  $E \rightarrow B$

- cylinder
- moebius strip

- Torus

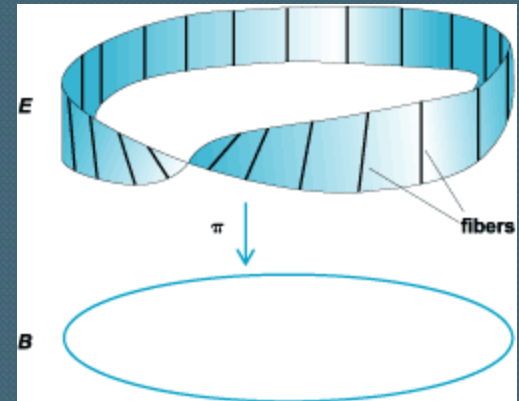
- circle bundle on circle

- Vector Bundle

- Earth's electromagnetic field.

- Spinor bundle – fiber is spinor.

- Frame bundle – fiber is frame (coord system)



# Bundle values

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- How fibers are glued together is the bundle structure.
- Fiber bundles all throughout advanced mathematics.
- Physics as functions on bundles over space-time.
- Space-time is the base manifold.

# Unification

---

- ◉ Want to finally glue General Relativity with the Standard Model
- ◉ This is a Theory of Everything (ToE)
- ◉ Need one more set of math abstractions
  - Super \*

# Need for non-commutating variables

---

- $x, y, z$  as functions on Euclidean space
- momentum, position as functions on particles
- QM and QFT need some non-commuting variables.
- Coords now usual space-time  $x, y, z, \dots$  and Grassmann valued spinors.

# Super-ness

---

- Super symmetry (bosons rotate into fermions)
- Super particles - hypothetical
- Super manifolds – manifold abstraction
- Super connections – connection abstraction
- Super strings – competing theory of everything

# Super math

---

- Need to redo:
  - Calculus – functions, derivatives, integration
  - Differential equations
  - Manifolds
  - Geometry
  - Topology
  - Physics
- To incorporate anticommuting variables

# Supermanifolds

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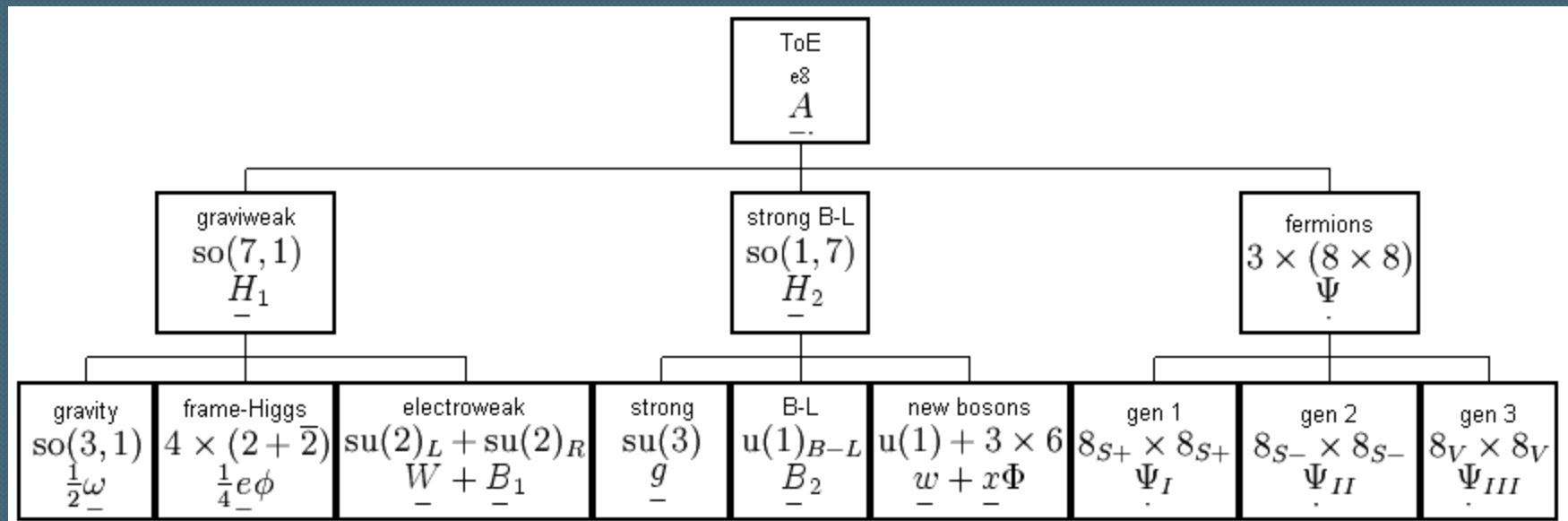
- Reconstruct manifold theory allowing some commuting variables (classical measurements) and some non-commuting variables (quantum measurements) by prefixing with “super”
- Coordinates now  $[x,y,z,\dots]$  and  $[\theta_1,\theta_2,\theta_3,\dots]$ 
  - Linear Algebra  $\rightarrow$  superlinear algebra
  - Manifold  $\rightarrow$  supermanifold
  - Connection  $\rightarrow$  superconnection
  - Group  $\rightarrow$  supergroup
  - Bundle  $\rightarrow$  superbundle

# Lisi Theory

---

- All **fields** of the **standard model** and **gravity** are unified as an **E8 principal bundle connection**. A non-compact real form of the **E8 Lie algebra** has **G2** and **F4 subalgebras** which break down to **strong su(3)**, **electroweak su(2) x u(1)**, **gravitational so(3,1)**, the **frame-Higgs**, and three generations of **fermions** related by triality. The interactions and dynamics of these **1-form** and **Grassmann** valued parts of an **E8 superconnection** are described by the **curvature** and **action** over a **four dimensional base manifold**.
- **Exceptionally Simple Theory of Everything**

# Details



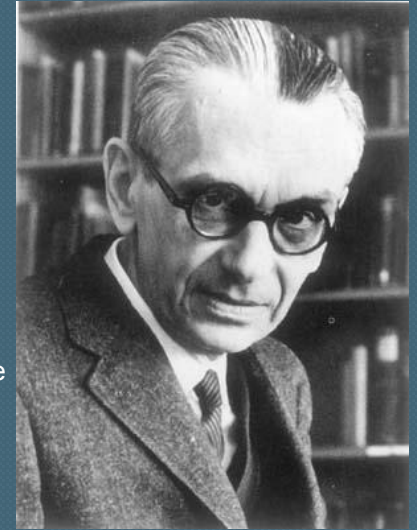
# Consensus?

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- Not clear, although not looking good
  - Some very technical details, but perhaps fixable.
- However, it took Einstein 10+ years to get GR working correctly.

# Gödel and the End of Physics

- A few scientists claim **Gödel's incompleteness theorem** proves a TOE cannot exist.
- **Gödel's theorem**
  - any sufficiently complex mathematical theory that has a finite description is either inconsistent or incomplete.
- **Freeman Dyson** has stated that
  - “Gödel’s theorem implies that pure mathematics is inexhaustible. No matter how many problems we solve, there will always be other problems that cannot be solved within the existing rules. [...] Because of Gödel’s theorem, physics is inexhaustible too. The laws of physics are a finite set of rules, and include the rules for doing mathematics, so that Gödel’s theorem applies to them.”
- **Stephen Hawking** originally believed in the Theory of Everything but, after considering Gödel's Theorem, concluded that one was not obtainable.
  - “Some people will be very disappointed if there is not an ultimate theory, that can be formulated as a finite number of principles. I used to belong to that camp, but I have changed my mind.”
- **Douglas S. Robertson**
  - Conway's game of life counterexample: underlying rules are simple and complete, but there are formally undecidable questions about the game's behaviors.
- Analogously, it may (or may not) be possible to completely state the underlying rules of physics with a finite number of well-defined laws, but there is little doubt that there are questions about the behavior of physical systems which are formally undecidable on the basis of those underlying laws.



Kurt Gödel,  
1906-1978

# Questions?

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The End?